

# A comparative study of forecasting methods using real-life econometric series data

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## Abstract

**Paper aims:** This paper presents a comparative evaluation of different forecasting methods using two artificial neural networks (Multilayer Perceptron network and Radial Basis Functions Neural Network) and the Gaussian process regression.

**Originality:** Due to the current world scenario, solving economic problems has become extremely important. Artificial neural networks are one of the most promising tools to forecast economic trends and are being widely studied in economic analyses. Therefore, due to the concerns about the performance of different forecasting methods to solve economic problems, this study contributes with an example of the forecasting performance of artificial neural network models compared with Gaussian process regression using Nelson-Plosser and U.S. macroeconomic real-life data sets.

**Research method:** Two real-life data sets were used to evaluate the forecasting methods proposed in this paper. These data sets were normalised to values between zero and one. After that, the data training was performed and, once it was built, a model was used to generate forecasts. Thus, observations were made to verify how accurately the fitted model forecast the values.

**Main findings:** The results obtained from the study show that, for all forecasting horizons, multi-layer perceptron networks and Gaussian process regression models had the most satisfactory results. On the other hand, the radial basis functions neural network model was unsuitable for econometric data.

**Implications for theory and practice:** This study contributes to a discussion about artificial neural networks and Gaussian process regression models for econometric forecasting. Although artificial neural networks are mainly used in economic analyses, the results showed that not all models, such as radial basis functions neural networks, present good results. In addition, the regression of the Gaussian process showed promising results to forecast econometric data.

## Keywords

Artificial neural networks. Gaussian process regression. Forecasting. Macroeconomic series.

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## 1. Introduction

Over the last few years, the world has been going through several cascading crises, and economic problems have become increasingly concerning in many countries. COVID-19 affected many aspects of society, from the economic to the digital spheres (Robinson et al., 2021). Thus, studies that can contribute to helping governments forecast their indicators and recover their economy are in growing demand. Several studies are developing new theories, methods, and tools, such as neural networks, to improve the modelling of economic problems (Khraisha, 2020).

Time series are observations on economic variables, which can be drawn from various fields of economics and business. Such variables include: dividend price ratio, dividend yield, industrial production growth, treasury



bill rate, inflation, long-term return, earnings-price ratio, default yield spread, default return spread, volatility of industrial production growth, volatility of producer's price index (Chen et al., 2016) stock market indices, unemployment rates, and market shares. Out-of-sample forecasts for such variables are often needed to set policy targets. For example, the forecast of a market share company in the next few months may lead to changes in the budget allocation for advertising.

The quantitative analysis of these phenomena can be called econometric time series models. Econometrics is a statistical method applied to economic data to give empirical content to economic relationships (Brooks et al., 2019).

Forecasting methods linked to economic problems are used to predict economic variables of political debate in many countries. Some examples include: the projection of the future unemployment rate, which defines the condition of a country's economic equilibrium and is fundamental to social development (Dritsakis & Klazoglou, 2018); the industry volatility forecasting, crucial to many essential issues in finance (Chen et al., 2016); the prediction in measuring equity risk premiums, perhaps the most widely studied problem in finance (Gu et al., 2020); and the forecast of variations in the price of products and services in agribusiness, which is an essential part of the gross domestic product of Brazil (Puchalsky et al., 2018). The examples cited above highlight an important question about which model is the most accurate to help political decisions.

A method that has been widely studied in economic analysis is Artificial Neural Networks (ANN). ANN are computational systems that can be implemented in software or hardware under the influence of biological research regarding the human brain (Puchalsky et al., 2018). Many researchers agree that ANN is the best predictor and the best performing non-linear analysis method (Gu et al., 2020). The ANN architectures used in the field of economics are Backpropagation and Radial Basis Function Networks (RBF).

Thus, several authors and researchers face concerns about the capability of different forecasting methods to solve economic problems. Some studies compared ANN with traditional forecasting techniques (Gu et al., 2020; Puchalsky et al., 2018; Safari et al., 2016; Yu et al., 2006; Zhang et al., 2019).

In general, ANN outperformed these techniques, and was able to capture dynamic non-linear trends and seasonal patterns and the interactions between them. Therefore, this paper presents a comparative study of the performance of the econometric model and ANN. However, what sets this study apart from many others published in the area is that it uses the Nelson-Plosser (Nelson & Plosser, 1982) and U.S. macroeconomic (Smets & Wouters, 2005) real-life data sets to analyse and evaluate both methods. Both data sets comprise fourteen U.S. data observed yearly, which generate fourteen economic time series widely used by researchers in this field. Furthermore, unlike most other studies in the field, this uses Gaussian Process Regression (GPR), which is capable of yielding reliable out-of-sample predictions in the presence of highly non-linear unknown relationships between dependent and explanatory variables.

Regarding the remainder of this paper, section 2 delineates some related works in which econometrics models were compared to ANN. Section 3 consists of a brief background review of forecasting methods, and section 4 presents the accuracy methods used to evaluate the performance of the forecasting methods. The experimental evaluation, final results, and some discussions are described in section 5. Finally, section 6 comprises the conclusion of this paper and suggestions for future work.

## 2. Related works

As mentioned previously, forecasting techniques are fundamental in several fields of research. Several authors explored these techniques to demonstrate which method has better performance. In this section, some comparative works are presented.

A systematic review was performed in indexed journals and present in one of the following databases: Web of Science, Scopus or Academic Search Premier (ASP – EBSCO). The inclusion criteria for articles were as follows: containing one or more of the chosen descriptors (i.e., <forecasting>, <method forecasting>, <time series forecasting> or <economic time series>); published in English, between 2016 and 2021. The exclusion criteria were conference abstracts or book chapters.

The first study (Puchalsky et al., 2018), aims to evaluate the performance of Wavelet Neural Networks (WNN), combined with five optimisation techniques to obtain the best time series forecast, which have been used in two case studies on the agribusiness sector. The performance of the optimisation techniques when training the WNN was compared to the well-established Backpropagation algorithm and Extreme Learning Machine (ELM), assuming accuracy measures. In both cases analysed in the study, ELM outperforms most of the methods during validation procedures. However, during short and long-term tests, metaheuristic optimisation methods used for training outperformed ELM results in almost all cases.

A time series study regarding quarterly observations on Gross Domestic Product (GDP) compares the results of a multi-layer perceptron network with those of the Autoregressive Integrated Moving Average (ARIMA) and regression as benchmark methods (Safi, 2016). Using Root Mean Square Error, the empirical results show that ANN performs better than the traditional methods in forecasting GDP.

The work of (Bandeira et al., 2020) proposes a forecasting application strategy considering two procedures: the combination of state-of-the-art forecasting methods and the selection of forecasting methods based on the accuracy of the models. The authors propose two combination strategies: simple mean and weighted mean based on the accuracy of the methods. They used two data sets with different characteristics – a public dataset of the competition and a private data set of spare part demand from an elevator industry. The results showed that the combination of forecasting methods is valuable if a weighting scheme based on the performance is employed. However, the combination using a simple mean outperforms other forecasting methods.

In another study, (Zhang et al., 2019) presents a comparison of the performance of two forecasting techniques. The authors compared the econometric and computational approaches of ANN algorithms, and the result shows that the ANN approach predicts the most accurate weekly and monthly data. On the other hand, econometric forecasting models produce better one-step-ahead predictions than ANN-based algorithms using daily data.

Finally, (dos Santos et al., 2020) explores the use of integrated tools, aiming at the creation of a continuous decision aid system, a Digital Twin, using the Discrete Event Simulation to describe and optimise the behaviour of a process, with the aid of forecasting models based on the moving average, single exponential smoothing, and double exponential smoothing methods. The proposed approach was applied to a real subject, which concerns a material supply process in Kanban stations of an aeronautical industry. The results of both techniques showed the great versatility of simulation and forecasting methods regarding their use in an integrated way, forming a Digital Twin.

In general, the studies present a comparative evaluation of forecasting methods applied in different contexts and the results on each one's performance. Nevertheless, to the best of our knowledge, no study has ever compared artificial neural networks and Gaussian process regression performance using Nelson-Plosser (1860–1970) and U.S. macroeconomic (1947–2009) real-life data.

### 3. Forecasting methods

Forecasting methods predict future values based on a given time series dataset by evaluating historical data and making assumptions on future trends. This can be applied to many areas of the decision-making process, such as operations management, risk management, economics, industrial process control, and demography (dos Santos et al., 2020).

Forecasting is a significant problem spanning many fields, including business and industry, government, economics, environmental sciences, medicine, social science, politics, and finance. Forecasting problems are often classified as short-term, medium-term, and long-term. Short-term forecasting problems involve predicting events within a short time span (days, weeks, and months). Medium-term forecasts can extend from 1 to 2 years into the future, and long-term forecasting problems can extend far beyond that (Montgomery et al., 2015).

In this study, econometric modelling methods are used to forecast a long-term and medium-term horizon using Nelson-Plosser and U.S. macroeconomic series data.

#### 3.1. Gaussian process regression

Gaussian process regression (GPR), a new machine learning regression method developed in recent years, is a non-parametric model algorithm based on a Bayesian network (Rasmussen & Nickisch, 2010). The GPR algorithm can adaptively determine the number of model parameters according to the information provided by training samples, add prior knowledge of the existing objects into the modelling process, and then combine the actual experimental data to obtain the posterior Gauss process model (Fu et al., 2019).

GPR has been applied in various fields due to many desirable properties, such as the existence of explicit forms, the ease of obtaining and expressing uncertainty in predictions, the ability to capture a wide variety of behaviour through covariance functions, and a natural Bayesian interpretation (Wu & Wang, 2018). As an example, GPR has been used to capture randomness in wind energy, since wind variability is stochastic (Yan et al., 2016).

GPR can be considered a Bayesian non-parametric approach to regression, where the function from the Gaussian processes takes values in a function space (Ballabio et al., 2019). A Gaussian process is a stochastic

process expressed through the mean and the covariance (Rasmussen & Nickisch, 2010). GPR assumes that the output  $y$  of a function  $f$  with input  $x$  can be expressed as:

$$y = f(x) + \varepsilon, \tag{1}$$

where,  $y = [y_1, y_2, \dots, y_n]^T$  is the  $n \times 1$  dimensional observation vector affected by noise,  $x = [x_{i,1}, x_{i,2}, \dots, x_{i,L}]^T$ ,  $x_i \in R^L$ ,  $i = 1, 2, \dots, n$  is  $n \times 1$  dimensional random variable obeying Gaussian distribution;  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$  is the observation noise vector obeying Gaussian distribution independently of each other,  $N$  is normal distribution and  $\sigma_\varepsilon^2$  is the error variance.

Gaussian processes (GP) for regression prediction are a supervised machine learning problem that can learn to map the relationship between input and corresponding output values when given the training set composed of input-output pairs (Fu et al., 2019). The GPR method first defines a prior distribution in the function space for Bayesian inference, and by learning from the test sample data, it can describe the  $f(x)$  indirectly and accurately (Fu et al., 2019).

The GPR is based on GP modelling, where GP is a set of joint Gaussian distributions of arbitrary finite random variables (Ballabio et al., 2019; Fu et al., 2019), and its statistical properties are uniquely determined by the mean and covariance functions. In GPR,  $f(x)$  is distributed as a Gaussian process:

$$f(x) \sim GP(\mu(x), k(x, x^*)), \tag{2}$$

where  $f(x)$  is defined by its mean  $\mu(x)$  and covariance  $k(x, x^*)$ ;  $x^*$  is the estimated value; and the variances  $\sigma^2$  for the elements of  $f(x)$  can be obtained from the diagonal of the covariance matrix.

Let the observation data set  $T = \{(x_i, y_i) | i = 1, 2, \dots, n\}$ , where  $T$  also represents the training sample set or the learning sample set. Under the condition that the training set  $T$  has been obtained, the posterior distribution of the predicted value  $y^*$  is as follows:

$$p(y^* | T, x^*) \sim N(\mu(y^*), k(x, x^*)) \tag{3}$$

The covariance function  $k$ , also known as the GPR kernel, and models the dependence of the function values between different values of  $x$ . In this study, the kernel function chosen was the squared exponential covariance function, which is defined as:

$$k(x_i, x_j; \theta) = \sigma_f^2 \exp \left[ -\frac{1}{2} \sum_{l=1}^L d_l (x_i^l - x_j^l)^2 \right] + \sigma_n^2 \delta_{ij}, \tag{4}$$

where  $x_i^l$  is the component of the input vector  $x_i \in R^L$  in the  $l$  dimension;  $\theta = \{\sigma_f^2, D, \sigma_n^2\}$  is the hyperparameter set of the kernel function, which is a set of vectors consisting of signal variance  $\sigma_f^2$ , noise variance  $\sigma_n^2$ , and model covariance function parameters;  $D = \text{diag}(d_1, \dots, d_1, \dots, d_L)$  is a symmetric matrix, and  $d_l$  reflects the degree of association between the input variable and the target output variable;  $\delta_{ij}$  is the Kronecker operator.

The GPR model's building and testing processes are shown in Figure 1. In this study, the forecasting based on the GPR method can be divided into the GPR training model construction stage and the test stage.

In general, the GPR training process is based on the Bayesian principle to obtain the maximum posterior likelihood estimate of  $\theta$  as the hyperparameter optimal solution. When the training of GPR is completed, the corresponding covariance matrix can be obtained and  $x^*$  to predict, then the  $y$ -value can be obtained.

### 3.2. Artificial neural network

Artificial Neural Networks, which were originally inspired by research on the human brain, are a computational method that can be implemented in hardware or software (Puchalsky et al., 2018; Samadianfard et al., 2020). They have been used extensively in different application areas, especially for non-linear time series modelling (Chen et al., 2020; Li et al., 2019; Sun et al., 2019). ANN have several advantages over other forecasting models, such as the capacity of fitting a complex non-linear function (Büyükkahin & Ertekin, 2019). ANN's flexibility and non-linear learning capabilities make this method coherent with research on forecasting (Gupta et al., 2017).

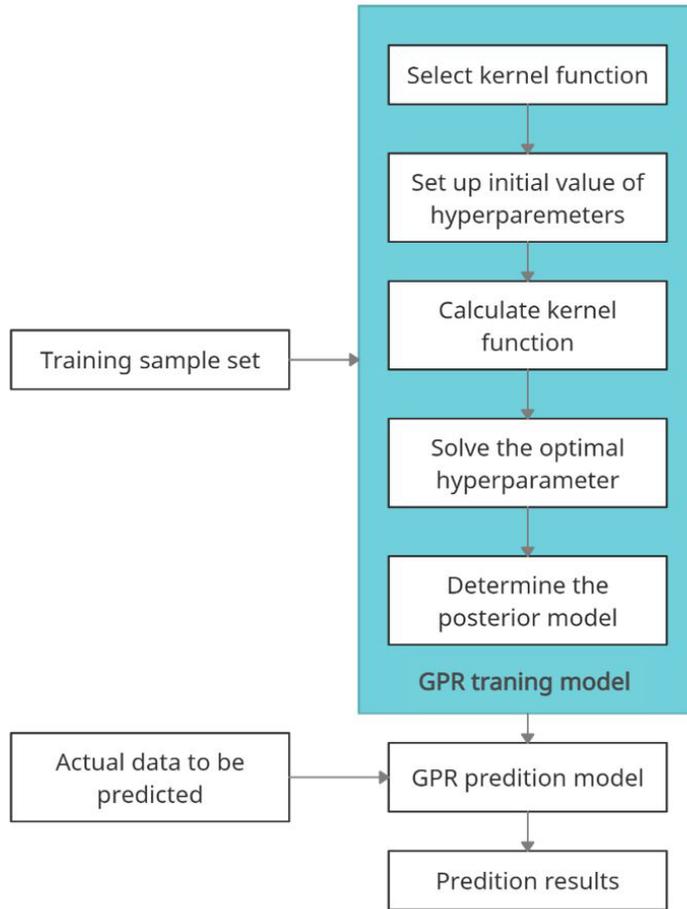


Figure 1. GPR modelling and testing process. Source: (Fu et al., 2019).

The ANN, as a prevalent modelling method, has been used for the identification of the complicated non-linear relationship of inputs and output (Nourani et al., 2021).

ANN provides a flexible computation framework for non-linear modelling in several applications, and thus, the number of layers and the neurons at each layer can easily vary. Moreover, ANN does not demand any prior assumption, such as input data stationarity, and the characteristics of the data largely determine an ANN configuration (Büyüksahin & Ertekin, 2019).

ANN combines several processing layers, using simple elements operating in parallel. It consists of an input layer, one or more hidden layers, and an output layer. Each layer contains several neurons that can modify the inputs with weights and activation functions to obtain the output. In this study, a three-layered structure composed of (i) input layer, (ii) hidden layer, and (iii) output layer is used, as seen in Figure 2.

The mathematical formulation of ANN models can be expressed by Equation 5. In this Equation, at any given time  $t$ ,  $w_{ij}$  and  $w_j$  are model weights,  $w_0$  is the threshold (bias),  $H$  and  $N$  are the number of hidden and input nodes, respectively, and  $e_t$  is a noise or error term.

$$y_t = w_0 + \sum_{j=1}^H w_j f \left( w_{0j} + \sum_{i=1}^N w_{ij} y_{t-1} \right) + e_t \quad (5)$$

Some reassuring types of ANN techniques are applicable in engineering problems, such as multi-layer perceptron and radial basis function (Sadeghi et al., 2021), both of which shall be briefly expounded in the following sections.

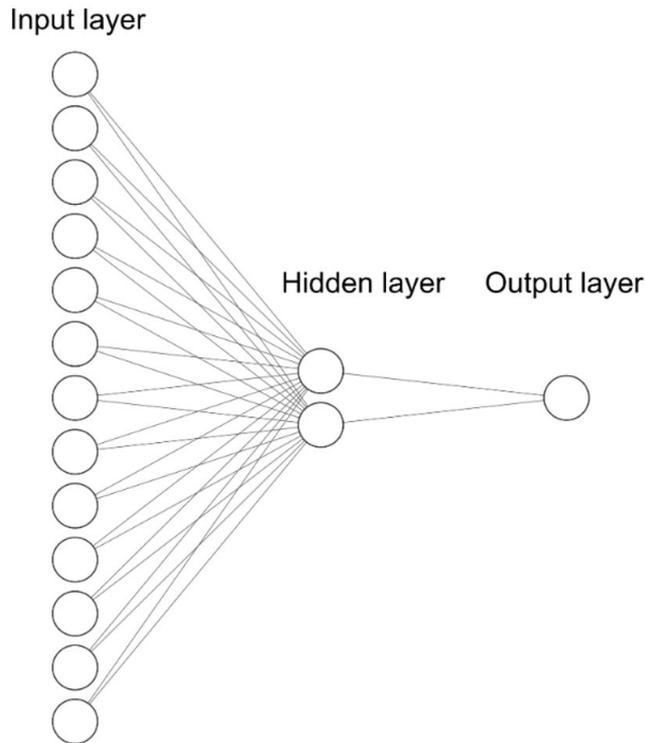


Figure 2. Neural Network architecture. Source: Authors.

### 3.2.1. Multi-layer Perceptron network

The multi-layer perceptron network (MLP), also called the Backpropagation network, is a feedforward neural network with a supervised learning rule to search for weight values employing a linear activation function – an approach that tends to solve complex problems (Madhiarasan & Deepa, 2017).

These networks learn linear and non-linear relationships between the input and output vectors because of hidden layer neurons and their non-linear transfer function. The most commonly used non-linear transfer function is the hyperbolic tangent sigmoid activation function applied over the net input of the hidden layer to obtain the respective output. The backpropagation gradient descent is employed to train multi-layer perceptron networks. These networks are fully connected, which induces the faster convergence of the network (Madhiarasan & Deepa, 2017). MLP is a supervised learning algorithm that learns a non-linear function and maps inputs to outputs by training on a dataset (Feng et al., 2020). The topology of the MLP neural network includes an input layer, one hidden layer, and an output layer, and the operations can be divided into two steps: feedforward and backpropagation. In the feedforward step, an input pattern is applied to the input layer, and its effect propagates, layer by layer, through the network until the output is produced. The network current output value is then compared to the expected output, and an error signal is computed for each of the output nodes.

### 3.2.2. Radial basis functions neural network

A radial basis function neural network (RBF) is a feedforward neural network that uses a radial basis function as its activation function. The connections between the input and hidden layers are not weighted, and the transfer functions on the hidden layer nodes are radial basis functions in the RBF, which is different from those in BPNN and generally train faster than MLP due to the use of the radial basis functions (Zhang et al., 2013).

The RBF is a general class of non-linear and three-layer feedforward neural networks: (i) an input layer, (ii) a hidden layer with neurons, and (iii) an output layer with one or several nodes (Rani R. & Victoire T, 2018). Yet, instead of using a sigmoid activation function, it performs a radially symmetric linear combination of  $n$  basis functions around a centre. Formally, for a given input  $x$ , the network output  $y$  can be written as:

$$y = \sum_{i=1}^n \omega_i R_i(\mathbf{x}) + \omega_0 \quad (6)$$

where  $\omega_i$  are weights,  $\omega_0$  is a bias term,  $n$  denotes the number of the neurons in the hidden layer, whereas  $R_i$  are the activation functions, given by:

$$R_i(\mathbf{x}) = \varphi(\|\mathbf{x} - c_i\|) \quad (7)$$

where  $\varphi$  is the radial function providing the non-linear feature of the model, and  $c_i$  represents the so-called RBF centres.

The most popular RBF is given by the Gauss function:

$$\varphi(r) = \exp(-r^2 / \sigma^2) \quad (8)$$

with  $r$  indicating the Euclidean distance between the input vector  $\mathbf{x}$  and centre  $c_i$  and  $\sigma$  being the so-called spread parameter to be determined.

## 4. Methodology for data analysis

This section describes the forecasting procedure to develop the ANN and GPR models using two real data sets for the study.

### 4.1. Data set

Two data sets were considered for the forecasting evaluation: the first study was the Nelson-Plosser (Table 1), and the second was the U.S. macroeconomic time series (Table 2). Both included fourteen-time series.

The Nelson-Plosser was a U.S. macroeconomic time series that used econometric models dynamically through regressions models, and is currently described as one of the most advanced time series for forecasting. Nelson-Plosser is an annual time series and includes variables such as Real Gross National Product (GNP), Stock Prices, Real Money, and the Unemployment Rate.

The second data set comprises fourteen U.S. macroeconomic data set includes fourteen-time series updated quarterly, available from Jan. 1947 to Jan. 2009. This model comprises a more recent data series if compared to the classic Nelson-Plosser database. Despite having more recent data on the North American economy, the study was limited to the database available in the Federal Reserve Bank of St. Louis (FRED, 2021).

This study considered the Consumer Price Index (CPI) as forecasting data (highlighted in grey in Tables 1 and 2) and defined the other data as inputs sets to adjust the predictor model.

The original CPI data from Nelson-Plosser covered the years 1860 through 1970, as shown in Figure 3a, and CPI from U.S. macroeconomic covered the years 1947 to 2009, as shown in Figure 3b.

The Consumer Price Index, also called cost-of-living index, measures the average change over time in the prices paid by urban consumers for a representative basket of consumer goods and services, including everything from food items to automobiles to rent (Konny, 2020). As the most widely used measure of inflation, the CPI is an indicator of the effectiveness of government policies.

Figures 4 and Figure 5 show the graphs of each inputs time series for both data sets.

### 4.2. Qualitative analysis

This paper proposes and analyses the use of a selection strategy for choosing the best forecasting model based on the performance of forecasting accuracy measures. There is no specific rule governing the data split in the literature. However, it is generally agreed that most data points should be used for model building (Qi & Zhang, 2008). The selected model is designed using the training intervals and is evaluated on the testing interval for the long-term and medium-term to analyse its performance on future samples.

Three scenarios were proposed for this study. The first and second long-term prediction scenarios spanned 10 and 5 years, and in the third case, a medium-term prediction of 2 years was used for the two data sets. Since U.S. macroeconomic time series are quarterly, four observations cover one year.

Table 1. The Nelson-Plosser data set (1860-1970).

Initials	Time series	Coverage	Length
GNPR	Real Gross National Product	1909-1970	62
GNPN	Nominal GNP	1909-1970	62
GNPPC	Real per capita GNP	1909-1970	62
IPI	Industrial production	1860-1970	111
E	Employment	1890-1970	81
UR	Unemployment rate	1890-1970	81
GNPD	GNP deflator	1889-1970	82
CPI	Consumer price index	1860-1970	111
WN	Wages	1900-1970	71
WR	Real wages	1900-1970	71
MS	Money stock	1889-1970	82
MV	Velocity	1869-1970	102
BY	Bond yield	1900-1970	71
SP	Common stock prices	1871-1970	100

Source: Authors.

Table 2. U.S. macroeconomic series, 1947-2009.

Initials	Time series	Coverage	Length
COE	Paid compensation of employees in \$ billions	Q1 1947-Q1 2009	249
CPI	Consumer price index	Q1 1947-Q1 2009	249
FEDFUNDS	Effective federal funds rate	Q2 1954-Q1 2009	219
GCE	Government consumption expenditures and investment	Q1 1947-Q1 2009	249
GDP	Gross domestic product	Q1 1947-Q1 2009	249
GDPDEF	Gross domestic product price deflator	Q1 1947-Q1 2009	249
GPDI	Gross private domestic investment	Q1 1947-Q1 2009	249
GS10	Ten-year Treasury bond yield	Q3 1954-Q1 2009	224
HOANBS	Non-farm business sector index of hours worked	Q1 1947-Q1 2009	249
M1SL	M1 money supply (narrow money)	Q4 1958-Q1 2009	201
M2SL	M2 money supply (broad money)	Q4 1958-Q1 2009	201
PCEC	Personal consumption expenditures in \$ billions	Q1 1947-Q1 2009	249
TB3MS	Three-month treasury bill yield	Q1 1947-Q1 2009	249
UNRATE	Unemployment rate	Q1 1948-Q1 2009	245

Source: Authors.

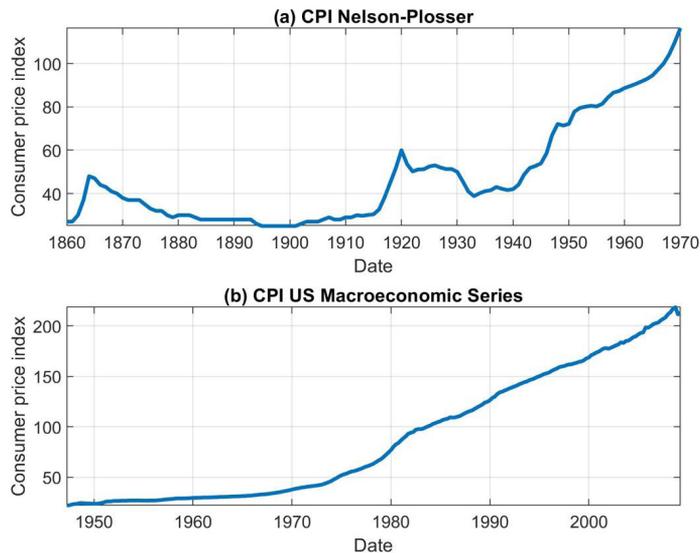


Figure 3. Consumer Price Index. Source: Authors.

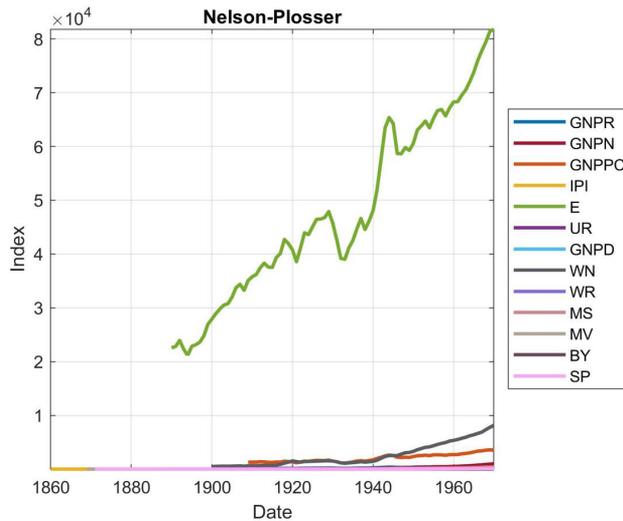


Figure 4. Nelson-Plosser. Source: Authors.

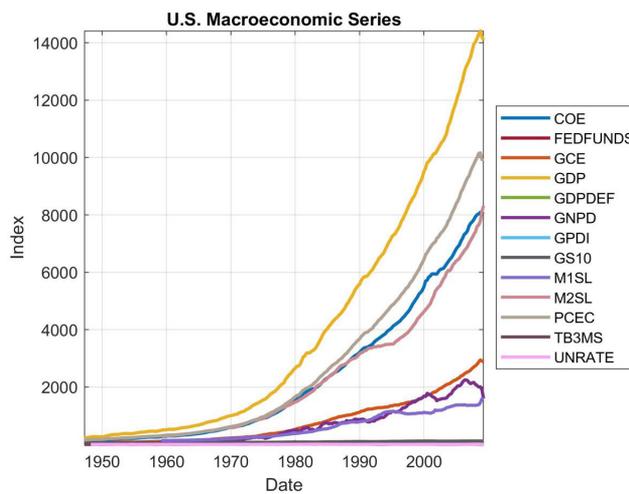


Figure 5. U.S. Macroeconomic Series. Source: Authors.

### 4.3. Pre-processing

The data were normalised to values between zero and one, and the missing values were replaced by the mean value of each variable series for the data sets. Normalisation was necessary to ensure that values measured on different scales were standardised.

### 4.4. Software

The algorithms were developed using the Matlab® R2020a software in a 4 GB RAM computer, with a 3.41 GHz Intel® Core processor and Windows® 32-bit operating system.

## 5. Experimental evaluation

This section formally compares the forecasting capabilities of the proposed GPR, MLP, and RBF. The comparison among the techniques was based on the most widespread mean value measures for forecasting, i.e. (a) the Mean

Squared Errors (MSE); (b) The Root Mean Squared Errors (RMSE); (c) the Mean Absolute Error (MAE); and (d) the Mean Absolute Percentage Errors (MAPE).

## 5.1. Forecasting accuracy measurements

The first measurement, MSE, is the average of the squared error. Since these prediction errors are squared in the MSE calculation, it significantly influences larger errors. Therefore, this property makes MSE worthwhile when significant errors are not wanted but penalise outliers (Büyükhahin & Ertekin, 2019). Equation 9 shows the MSE metric, where  $e_t = y_t - \hat{y}_t$  and  $y_t$  is the data value,  $\hat{y}_t$  is the forecasted value at time  $t$ , and  $n$  is the forecasting horizon.

$$MSE = \frac{1}{n} \sum_{t=1}^n e_t^2 \quad (9)$$

The RMSE, also known as the root mean square deviation (RMSD), is a quadratic scoring rule that measures the average magnitude of the error. It is the square root of the average squared differences between prediction and actual observation, Equation 10.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n e_t^2} \quad (10)$$

The RMSE statistic provides information about the performance of a model by allowing a term-by-term comparison of the actual difference between the estimated and the measured value. The smaller the RSME value, the better the model's performance.

The MAE method measures the average magnitude of errors in a set of predictions without considering their direction. It is the average of absolute differences between prediction and actual observation over the test sample where all individual differences have equal weight, as seen in Equation 11.

$$MAE = \frac{1}{n} \sum_{t=1}^n e_t \quad (11)$$

The mean absolute percentage error is the percentage equivalent of MAE. Equation 12 shows the MAPE:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{e_t}{y_t} \right| \times 100 \quad (12)$$

## 5.2. Forecasting results and discussions

### 5.2.1. Long-term estimate (ten-period-ahead forecast)

In the first scenario, classified as long-term (Montgomery et al., 2015), MLP and GPR methods had good performances for a ten-period-ahead forecast horizon, since they obtained good MAPE results. Consequently, the MSE, RMSE and MAE values also indicate good performance for these models. However, the RBF model did not show satisfactory results compared to the others methods, as shown in Table 3.

In this case, if we assess only the RMSE value, the RBF model could be a possible solution for a ten-period-ahead forecast horizon. However, when analysing the MAPE value, the RBF model presents a high percentage of forecasting errors. Therefore, comparing all models, the MLP and GPR models are the most suited for this case.

In an investigation for long-term forecast horizons (Torra & Claveria, 2017), the authors found that the RBF network surpasses the GPR model, unlike the results of our study. Thus, in the first scenario, the three models had a good forecasting accuracy, but GPR outperforms the ANN models.

Figure 6 shows the results obtained by different methods for a ten-period-ahead forecast horizon. For better visualisation, the graphics below show the ten years prior to the prediction.

The two other scenarios that were subjected to the forecast are described in the following sections.

Table 3. Results of the ten-period-ahead forecast.

Forecasting Method	Time series	MSE	RMSE	MAE	MAPE (%)
RBF	Nelson-Plosser	0.191	0.437	0.344	39.663
	U.S. macroeconomic	0.142	0.377	0.343	38.435
MLP	Nelson-Plosser	0.010	0.102	0.077	8.857
	U.S. macroeconomic	0.005	0.072	0.054	6.021
GPR	Nelson-Plosser	0.004	0.065	0.038	4.121
	U.S. macroeconomic	0.000	0.014	0.012	1.339

Source: Authors.

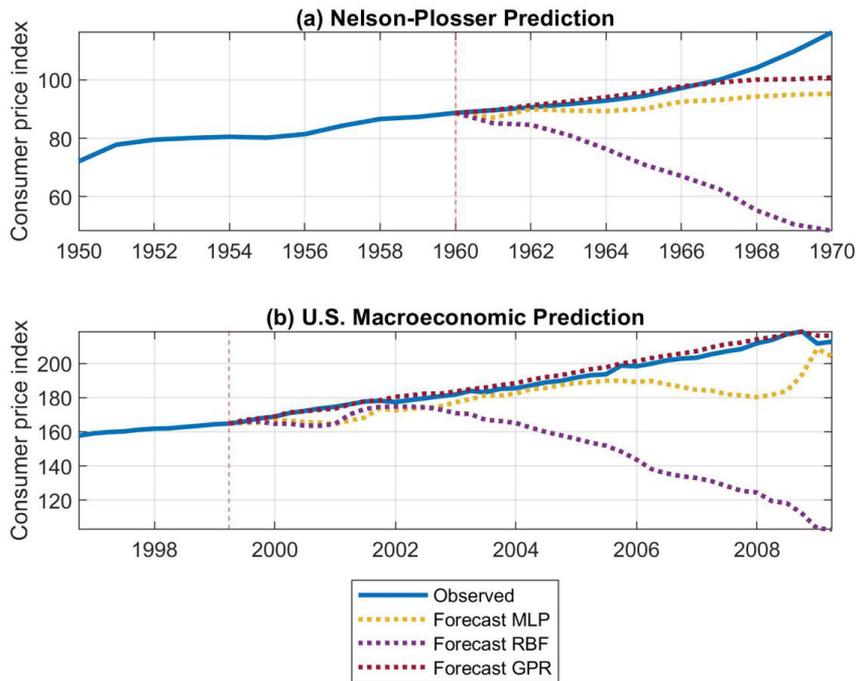


Figure 6. Results of the ten-period-ahead forecast. Source: Authors

### 5.2.2. Long-term estimation (five-period-ahead forecast)

In another long-term scenario, the results obtained for a five-period-ahead forecast horizon showed that the MLP and GPR models presented good accuracy values. In contrast, the RBF model once again did not show a satisfactory performance, similar to the first study. Table 4 shows the results.

Figure 7 shows the graphic sample forecast for a five-year horizon. For better visualisation, the graphics show ten years before starting the prediction. As in the first scenario, the performance of MLP and GPR was good, with GPR performing slightly better than MLP. However, in this second scenario, MLP had better performance than GPR.

Similar to the first scenario, the RBF model did not present a satisfactory result. This time the model had a worse performance, presenting a MAPE value quite discrepant in comparison with the other two models. This result shows that not all neural network models outperform the regression model in a medium-term horizon.

### 5.2.3. Medium-term estimation (two-period-ahead forecast)

The third scenario, classified as medium-term (Montgomery et al., 2015). The forecasting quality can also be observed when analysing the two-period-ahead forecast. These results are present in Table 5.

Table 4. Results of the five-period-ahead forecast.

Forecasting Method	Time series	MSE	RMSE	MAE	MAPE (%)
RBF	Nelson-Plosser	0.296	0.544	0.512	57.899
	U.S. macroeconomic	0.309	0.556	0.537	57.894
MLP	Nelson-Plosser	0.007	0.082	0.059	6.322
	U.S. macroeconomic	0.000	0.016	0.014	1.541
GPR	Nelson-Plosser	0.008	0.089	0.065	6.970
	U.S. macroeconomic	0.001	0.039	0.036	3.887

Source: Authors.

Table 5. Results of the two-period-ahead forecast.

Forecasting Method	Time series	MSE	RMSE	MAE	MAPE (%)
RBF	Nelson-Plosser	0.407	0.638	0.631	67.306
	U.S. macroeconomic	0.315	0.561	0.553	57.176
MLP	Nelson-Plosser	0.011	0.104	0.094	9.830
	U.S. macroeconomic	0.000	0.021	0.019	1.928
GPR	Nelson-Plosser	0.001	0.033	0.027	2.761
	U.S. macroeconomic	0.000	0.020	0.017	1.718

Source: Authors.

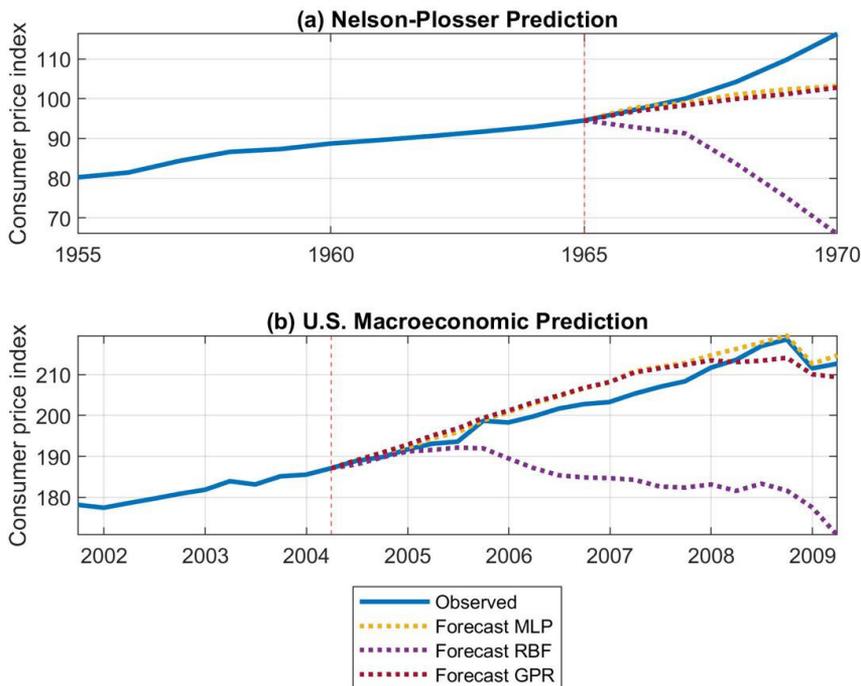


Figure 7. Results of the five-period-ahead forecast. Source: Authors.

Figure 8 shows the sample forecast graphically for a two-year horizon. For better visualisation, the graphics show the ten previous years before starting the prediction. According to predictive accuracy measures, the GPR is the best predictor, followed by MLP, similar to the previous scenarios.

This study implies that the MLP and GPR models are the most indicated for econometric data forecasts. The findings of this study complement ongoing research with the knowledge that, although ANN models are generally superior to regression models, both can be successfully applied for estimation (Safari et al., 2016; Zhang et al., 2019). These findings indicate that no particular model is the best in all situations (Zhang et al., 2019).

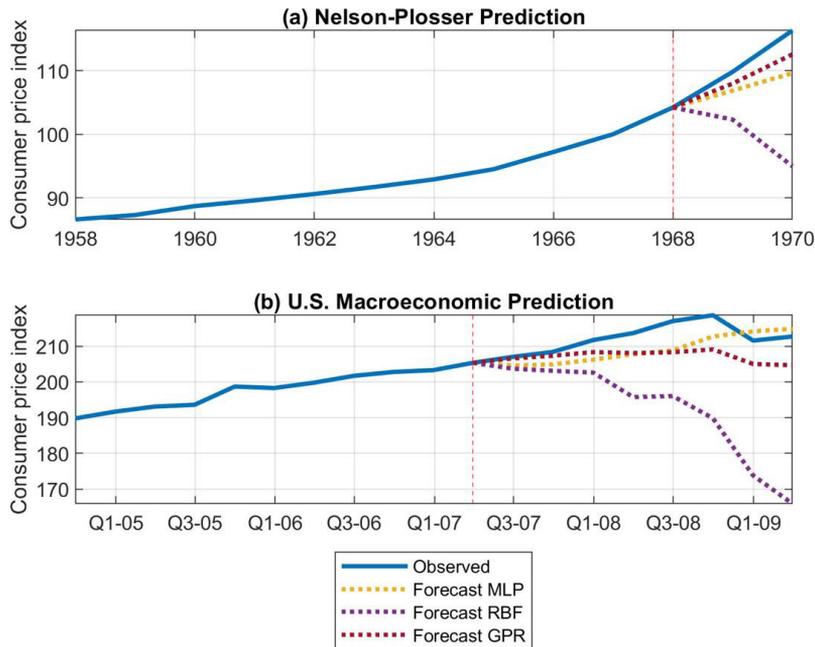


Figure 8. Results of the two-period-ahead forecast. Source: Authors.

Regarding the models used in this study (MLB, RBF and GPR), it is worth mentioning that the effect of parameters on the prediction accuracy is not discussed in this study. Better performance would be expected if some factors such as hyperparameters were further optimised and multiple techniques effectively integrated, which is the focus of our future research.

## 6. Conclusions

Considering the recent worldwide economic problems, ANN is one of the most promising tools to forecast trends and is widely studied in economics analysis.

Several authors voice concerns about the performance of different forecasting methods for solving economic problems. This study examines the forecasting performance of ANN models compared with GPR using econometric Nelson-Plosser and U.S. macroeconomic data sets.

The MLP model is the most extensively used neural network. In this study, the MLP showed similar results to GPR and presented a satisfactory performance for all forecasting horizons, indicating that MLP can provide a good forecasting accuracy for econometric data. In forecasts one and three, the GPR model showed good results and surpassed the ANN models, proving that it can also provide a good forecasting accuracy for econometric data.

The RBF model presented large error values for all forecast horizons. The high values in all metrics indicate this neural network is unsuitable for econometric data, reinforcing the current literature that no particular model is the best for all situations.

In future works, other forecasting methods should be tested and compared with ANN results when assessing whether ANN models outperform traditional econometric forecasting methods. The design of experiments should be used to delineate ANN and GPR hyperparameters. Another possible course for future studies is the development of a hybrid forecasting method that combines econometrics and ANN. We also suggest that future studies investigate whether the frequency of the series can influence the quality of predictions.

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