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# The Central Bank of Brazil's time-varying Taylor rule

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**Contents**



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real interest rate, Taylor rule, Kalman filter

**JEL Codes**
E43, F34

**Abstract** · Resumo

We estimate the Central Bank of Brazil's (BCB) monetary policy preferences over time by estimating a forward-looking Taylor-type rule through a state-space model with time varying parameters. We estimate two key parameters over time: the inflation parameter and the implicit inflation target. Our findings are: first, the BCB has over time taken a hawkish approach to inflation, over the 2003–2011 sample period, though since 2011 it has become more dovish. Second, BCB's implicit inflation target was largely on target, though until 2011 were below the center of the official target but have since stayed between the center and the upper band.

## 1. Introduction

Since 1999, the Central Bank of Brazil (BCB) has followed an explicit inflation-targeting (IT) mandate which, along with other economic reforms like the Real Plan in 1994, has contributed to a fall in inflation. Under this framework, the National Monetary Council (CMN) sets the annual inflation target. Since the start of the IT regime, the BCB has missed its target, and tolerance bands, five times: 2001, 2002, 2003, 2015, and 2017, requiring the respective BCB governors to draft an open letter, as mandated by presidential decree, to the finance minister.

This institutional characteristic of the BCB, among others, such as its newly-implemented *de jure* autonomy, have spared it from heavier criticism, namely that of political intervention, especially when compared with other emerging market

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central banks, but that is not to say the BCB has not been prone to scrutiny in its monetary policy preferences, especially given the abovementioned occurrences when it has missed its mandated inflation target.

This study estimates the BCB's monetary policy preferences over time by modeling a forward-looking Taylor-type rule through a state-space model with time varying parameters. The study focuses on estimating two key parameters: a) the Taylor rule's inflation parameter's behavior over time; and b) the Taylor rule inflation target over time, referred to as the "implicit inflation target". The purpose of the study is to assess the changes in the BCB's approach to inflation over the sample period and assess how the derived implicit inflation target has deviated from the official target. The latter point is critical, as though it is easy to compare annual inflation to the official target, the estimation of an implicit inflation target through a Taylor rule provides another tool to analyze a central bank's preference and performance *vis-à-vis* its target. Furthermore, the use of time-varying parameters has been an increasingly used method to address the Lucas critique of policy variation over time.

This study largely follows research by Aragón and Medeiros (2015) who estimate a forward-looking Taylor rule for the BCB with time-varying parameters, though unlike Aragon and Medeiros, our estimation keeps most parameters constant in time, with three different approaches to allowing the inflation parameter to vary. This study also updates the sample period studied by Aragon and Medeiros. Furthermore, a second paper this study follows is that of Leigh (2008) who estimated the implicit inflation target for the US Federal Reserve, and a subsequent study done by Klein (2012), who followed Leigh's approach to estimate the implicit inflation target for the South African Reserve Bank. Yet, different from Leigh and Klein, we adjust for endogenous regressors, in line with Kim (2004), as done by Aragón and Medeiros.

The main findings of this study are: 1) the BCB has over time taken a largely hawkish approach to inflation, following the Taylor principle over the 2003–2020 sample period, though with a fall in the inflation parameter starting in 2011—during what became known as a policy "U-turn" by the BCB, then under the governorship of Alexandre Tombini, where the BCB's monetary policy preferences were below the Taylor principle coefficient, recovering somewhat since, but not to the levels seen pre-2011; and 2) since the IT framework was adopted, the BCB's implicit inflation target has largely stayed within the bands set by the CMN, though the implicit targets were, until 2011, below the center of the official target but have since stayed between the center and the upper-band. The two conclusions are complementary and show a much more stimulative attitude by the BCB since 2011.

Section 2 presents the underlying methodology used to estimate the time-varying parameters. Section 3 presents the data used and unit root tests. Section 4 provides the estimation results, and section 5 provides concluding remarks on the study.

## 2. The empirical model

This section will outline the essence of the empirical model used to estimate the Taylor rule with TVPs. The rule assumes central banks respond to both inflation and output gap factors when setting monetary policy: when inflation goes above its target, the Taylor rule sets out a central bank must raise nominal interest rates by a larger proportion than the inflationary increase to control the rise in prices, the so-called "Taylor principle". The rule also stipulates central banks can lower nominal interest rates when output falls below potential.

Though Taylor's argued his rule applied to sample data from the Fed for 1987–1992, Clarida, Galí, and Gertler (1997) expanded on Taylor's original rule by adding a forward-looking specification, following the linear equation:

$$r_t^* = \bar{r} + \beta \left( E\left[\pi_{t,n} \mid \Omega_t\right] - \pi^* \right) + \gamma \left( E\left[y_t \mid \Omega_t\right] - y_t^* \right), \tag{1}$$

where $r_t^*$ is the optimal nominal interest rate; $\bar{r}$ is the equilibrium nominal rate in the long run; $\pi_{t,n}$ represents the inflation rate between period $t$ and $t+n$; $y_t$ is the output at period $t$; and $y_t^*$ is the potential output, which they define as the "level that would arise if wages and prices were perfectly flexible". Additionally, $\pi^*$ is the inflation target, whether explicitly set or implicit. Hence, $\beta$ and $\gamma$ are the coefficients for the inflation and output gap from their respective targets. Lastly, the expectation operator $E$ is added and $\Omega_t$ is the information set available to the central bank at period $t$. Assuming a consideration for the implied *ex ante* real interest rate, $rrt$, such that

$$rrt \equiv r_t - E\left[\pi_{t,n} \mid \Omega_t\right]. \tag{2}$$

Rearranging the terms above in (3) into (2), the optimal real interest rate, $rrt^*$, follows:

$$rrt^* = \overline{rr} + (\beta - 1) \left( E\left[\pi_{t,n} \mid \Omega_t\right] - \pi^* \right) + \gamma \left( E\left[y_t \mid \Omega_t\right] - y_t^* \right), \tag{3}$$

where the central bank aims for a real interest rate target which is a function of the neutral real interest rate $\overline{rr}$, as well as the deviations from the inflation target and potential output. Clarida et al. (1997) emphasize the importance of the magnitude of $\beta$ and $\gamma$, noting that if $\beta > 1$, the real interest rate targeted by the central bank serves to stabilize both inflation as well as output (given $\gamma > 0$), whereas if $\beta < 1$, the central bank's approach is more accommodative to inflation given inflationary spikes will not be met with a sufficiently large interest rate response.

Once again following the specification set by Clarida et al. (1997), an interest rate smoothing parameter is added to the Taylor rule. The justification of such a term is based on broad literature, namely Goodfriend (1991) which notes central banks smooth changes in interest rates to avoid large fluctuations which could in turn bring about turmoil in financial markets or hinder the central bank's credibility. Hence, a partial adjustment term is added to the Taylor rule specification:

$$rrt = (1-\rho)rrt^{*} + \rho rrt - 1 + \varepsilon_t, \tag{4}$$

Where $\rho \in [0,1]$, which captures the rate of smoothing in a central bank's interest rate decision and $\varepsilon_t$ is an exogenous random shock assumed to be i.i.d.[1]

Simplifying the output gap terms, $\left(E\left[y_t \,|\, \Omega_t\right] - y_t^{*}\right) = \tilde{y}_t$, and rearranging (5) into (4), we derive the Taylor-type rule used in our estimation:

$$rrt = (1-\rho)\left(\overline{rr} + (\beta - 1)\left(E\left[\pi_{t,n} \,|\, \Omega_t\right] - \pi^{*}\right) + \gamma \tilde{y}_t\right) + \rho rrt - 1 + \varepsilon_t. \tag{5}$$

We then adapt the Taylor rule in equation (4) into a state-space representation, with measurement and transition equations, following Commandeur and Koopman (2007), giving a Taylor rule with TVP specifications for the inflation parameter $\beta$, where $\beta$ follows a random walk process without drift. The random walk assumption for unobserved time-varying parameters is chosen given its widespread use in literature.[2]

$$\begin{aligned} rrt = (1-\rho)\left(\overline{rr} + (\beta - 1)\left(E\left[\pi_{t,n} \,|\, \Omega_t\right] - \pi^{*}\right) + \gamma \tilde{y}_t\right) + \rho rrt - 1 + \varepsilon_t, \\ \varepsilon_t \sim i.i.d.\,\mathcal{N}\left(0, \sigma^2\right); \end{aligned} \tag{6a}$$

$$\beta = \beta t - 1 + \upsilon_t, \qquad \upsilon_t \sim i.i.d.\,\mathcal{N}\left(0, \sigma_t^2\right). \tag{6b}$$

Similarly, a second specification of the model includes the output gap $\gamma$ as a time-varying parameter, alongside the time-varying $\beta$, hence

$$\begin{aligned} rrt = (1-\rho)\left(\overline{rr} + (\beta - 1)\left(E\left[\pi_{t,n} \,|\, \Omega_t\right] - \pi^{*}\right) + \gamma \tilde{y}_t\right) + \rho rrt - 1 + \varepsilon_t, \\ \varepsilon_t \sim i.i.d.\,\mathcal{N}\left(0, \sigma^2\right); \end{aligned} \tag{7a}$$

$$\beta = \beta t - 1 + \upsilon_{1t}, \qquad \upsilon_{1t} \sim i.i.d.\,\mathcal{N}\left(0, \sigma_{1t}^2\right); \tag{7b}$$

$$\gamma = \gamma_{t-1} + \upsilon_{2t}, \qquad \upsilon_{2t} \sim i.i.d.\,\mathcal{N}\left(0, \sigma_{2t}^2\right). \tag{7c}$$

Following Leigh and Klein, we also opt for a third specification, allowing the inflation target to vary in time and as done with the inflation parameter, using a random walk specification to estimate an implicit inflation target by the BCB, and comparing how

[1]Clarida et al. attribute a variety of interpretations which could cause such a shock, ranging from random components to policy, or potential for imperfect forecasts.

[2]A random walk process without drift has also been used by Kuzin (2006) for Bundesbank estimations, by Leigh (2005) for the US Federal Reserve, by Klein (2012) for the South African Reserve Bank, and by Laubach and Williams (2003) when estimating the natural real interest rate of the US.

monetary policy decisions have aligned with the explicitly defined target set by the CMN:

$$r_t = (1-\rho)\left(\overline{rr} + (\beta - 1)\left(E\left[\pi_{t,n} \mid \Omega_t\right] - \pi_t^*\right) + y\tilde{y}_t\right) + \rho r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.\,\mathcal{N}\left(0, \sigma^2\right), \tag{8a}$$

$$\pi^* = \pi t - 1^* + \upsilon_t, \qquad \upsilon_t \sim i.i.d.\,\mathcal{N}\left(0, \sigma_t^2\right). \tag{8b}$$

Following the method used by Leigh (2008), prior to estimating a Taylor rule through a Kalman filter in the state-space representation, initial parameters for the central bank's reaction function are estimated, serving as inputs for the Kalman filter estimation. These Taylor rule parameters are estimated using Ordinary Least Squares (OLS). To be sure, there is continuous debate on the potential for estimation bias in forward-looking Taylor rules given endogenous regressors. In this paper, we also estimated the initial Taylor rule parameters through Generalized Method of Moments (GMM) with lagged endogenous variables used as instruments but given there were no major differences in the final Kalman filter estimations, the OLS approach was opted for instead.

Furthermore, estimation of a Taylor rule with TVP using a state-space representation and a Kalman filter relies on the assumption that there is no correlation between regressors and the error term, with endogenous regressors providing for invalid inferences as noted by Kim (2004). In the case of forward-looking specifications of the Taylor rule, the expected inflation, and the output gap regressors are correlated with the error term $\varepsilon_t$, and hence the Kalman filter estimation is empirically inconsistent.

Kim (2004) and later Kim and Nelson (2006) propose a two-step Heckman-type (1976) procedure to correct biases in TVP estimations of a forward-looking Taylor rule, when endogeneity is present.

The two-step method proposed by Kim and Nelson follows a general state-space representation of a Taylor rule like the ones presented in (8a), (9a) and (10a):

$$r_t = (1-\theta)\left(\beta_{0,t} + \beta_{1,t}\pi_{t,J} + \beta_{2,t}g_{t,J}\right) + \theta_t r_{t-1} + e_t; \tag{9a}$$

$$\theta_t = \frac{1}{1 + \exp\left(-\beta_{3,t}\right)}; \tag{9b}$$

$$\beta_{i,t} = \beta_{i,t-1} + \varepsilon_{it}, \qquad \varepsilon_{it} \sim i.i.d.\,\mathcal{N}\left(0, \sigma_{\varepsilon,i}^2\right), \quad i = 0, 1, 2, 3; \tag{9c}$$

where $\beta_{0,t} = \beta_{0,t}^* - \beta_{2,t}\pi^*$ and $e_t = (1-\theta_t)\left[\beta_{1,t}\left(\pi_{t,J} - E_t\left(\pi_{t,J}\right)\right) + \beta_{2,t}\left(g_{t,J}\right)\right] + m_t$. In this specification, $r_t$ is the target nominal interest rate, $\beta_{0,t}$ is the neutral interest rate, $\pi_{t,J}$ is the expected inflation gap between $t$ and $t+J$, and $g_{t,J}$ is the output gap between periods $t$ and $J$. Under Kim and Nelson's specification, the smoothing parameter $\theta_t$ is also assumed to lie between 0 and 1. Kim and Nelson approximate the distribution of the error term $e_t$ by a GARCH(1,1) process:

$$e_t|\psi_{t-1} \sim \mathcal{N}\left(0, \sigma_{e,t}^2\right), \tag{10a}$$

$$\sigma_{e,t}^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{e,t-1}^2, \tag{10b}$$

Where $\psi_{t-1}$ is the information set going up to $t-1$.

The first step to estimate a bias-corrected forward-looking Taylor rule with TVPs is to obtain standardized forecast errors for an estimation of the endogenous regressors: inflation gap and output gap ($\pi_{t,J}$ and $g_{t,J}$ respectively in the above Kim and Nelson specification). The specification for the respective instrumental variable estimation is the following:

$$\pi_{t,J} = z_t'\delta_{1t} + v_{1t}, \qquad v_{1t} \sim i.i.d.\, \mathcal{N}\left(0, \sigma_{v_{1t}}^2\right); \tag{11}$$

$$g_{t,J} = z_t'\delta_{2t} + v_{2t}, \qquad v_{2t} \sim i.i.d.\, \mathcal{N}\left(0, \sigma_{v_{2t}}^2\right); \tag{12}$$

with

$$\delta_{it} = \delta_{i,t-1} + u_{i,t}, \qquad u_{i,t} \sim \mathcal{N}\left(0, \Sigma_{u,i}\right), \quad i = 1,2; \tag{13}$$

$$v_{j,t}^2 = a_{0j} + a_{1j} v_{j,t-1}^2 + a_{2j}\sigma_{v_{j,t-1}}^2, \qquad j = 1,2; \tag{14}$$

which suggest time-varying uncertainties in the parameters for inflation gap and output gap.

After some manipulations showed in the Appendix A, the error term in (11) to be rewritten as

$$e_t = \rho_1\sigma_e v_{1t}^* + \rho_2\sigma_e v_{2t}^* + \Omega_t, \qquad \Omega_t \sim \mathcal{N}\left(0, \left(1-\rho_1^2-\rho_2^2\right)\sigma_{e,t}^2\right), \tag{15}$$

whereby decomposing *et* into different components, $v_{1t}^*$ and $v_{2t}^*$, which are correlated with inflation gap and output gap, but uncorrelated with the error term $\Omega_t$, the endogeneity bias is corrected. Substituting (21) into Kim and Nelson's generic Taylor rule specification, we have:

$$r_t = (1-\theta)\left(\beta_{0,t} + \beta_{1,t}\pi_{t,J} + \beta_{2,t}g_{t,J}\right) + \theta_t r_{t-1} + \rho_1\sigma_e v_{1t}^* + \rho_2\sigma_e v_{2t}^* + \Omega_t,$$
$$\Omega_t \sim \mathcal{N}\left(0, \left(1-\rho_1^2-\rho_2^2\right)\sigma_{e,t}^2\right), \tag{16}$$

which is estimated using Maximum Likelihood Estimation via the Kalman filter.[3]

Finally, applying the Kim and Nelson Heckman-type two-step bias correction method to our notation of the Taylor rule measurement equations (8a), (9a), and (10a), which uses real interest rates, we have:

$$rr_t = (1-\rho)\left(\overline{rr} + (\beta-1)\left(E\left[\pi_{t,n}\,|\,\Omega_t\right] - \pi^*\right) + y\tilde{y}_t\right) + \rho rr_{t-1}$$
$$+ \nu_1\sigma_e v_{1t}^* + \nu_2\sigma_e v_{2t}^* + \Omega_t, \qquad \Omega_t \sim \mathcal{N}\left(0, \left(1-\nu_1^2-\nu_2^2\right)\sigma_{e,t}^2\right), \tag{17}$$

where $\nu_1$ and $\nu_2$ in (17) are equal to $\rho_1$ and $\rho_2$, respectively in (16).

[3]A more comprehensive derivation of the Kim and Nelson methodology is found in Appendix A.

## 3. Data

The study uses quarterly data from Q2 2003 to Q3 2020. The sample range was chosen because it contemplates a large portion of the time frame since the establishment of the BCB's inflation-targeting regime in 1999. The study starts in the second quarter of 2003 to mitigate volatility in the Taylor rule estimation coming from sharp interest rate and inflationary movements in the early 2000s, as well as avoid volatility from the October 2002 general elections.

The interest rate variable used in the Taylor rule estimation was the real swap rate: that is the natural log of the 360-day DI-fixed (*pré-fixado*) swap subtracted by the natural log of accumulated IPCA inflation expectations for the following 12-month period, collected by the BCB. The swap rate is opted in this case given a higher correlation with output than the Selic rate.

The output gap used is a weighted average of the utility capacity gap and the employment rate gap and the employment rate gap as defined in Alves (2001).

The inflation expectation series is the natural log of the BCB's survey of economic agents for the accumulated 12-month forward IPCA (*Índice Nacional de Preços ao Consumidor Amplo*, Ample Consumer Price Index) inflation index. The series is calculated daily for all forecasting institutions who have projected all the twelve months ahead of inflation, and the median of all contributors is taken. As noted, the monthly frequency of the BCB data is converted into a quarterly frequency by calculating the arithmetic mean for the three months in each respective quarter.

The natural log of the respective annual IPCA inflation targets set by CMN resolutions was used. For 2003 and 2004, when the respective inflation targets were changed from the original CMN resolutions, the latter inflation targets, set by the respective new CMN resolutions, were used: 4% with a 2.5-percentage point band for 2003, and 5.5% with a 2.5-percentage point band for 2004.

The instrumental variables used to estimate equations (13) and (14) include a constant term, one-to-four lags of the real interest rate (360-day DI-fixed swap discounted by 12-month forward IPCA inflation expectations), one-to-four lags of the 12-month forward IPCA inflation expectations, one-to-four lags of the output gap, one-to-four lags of the arithmetic mean for the quarter of industrial production as measured monthly by the IBGE.

Instrumental variables for the quarterly change in the exchange rate, the quarterly change in oil prices and the quarterly change in industrial production are also used. The first of the three, the *dlfx* variable is the percentage change in the quarterly exchange rate, where the quarterly exchange rate is the arithmetic mean for the quarter of the daily median exchange rate of Brazilian reais to one U.S. Dollar, taken from the Federal Reserve Bank of St. Louis' FRED database. The oil price variable *dloil* series follows the same method, but for the WTI crude daily oil price, taken from the U.S. Energy Information Administration database. The *dlpim* variable has the same construction, but for monthly industrial production, as surveyed by the IBGE.

Lastly, dummy variables are added for Q3 2005, Q2 2008 and Q2 2011 in the case of the instrumental variable regression for expected inflation (equation (11)), and a dummy for Q2 2020 for the instrumental variable regression for the output gap (equation (12)).

Unit roots tests of the variables are shown in the appendix. As seen in Table 4, the null hypothesis of a unit root is not rejected for the real interest rates ($rr_t$) and output gap ($\gamma$) series in when standard unit root tests are conducted. Yet, breakpoints are added, the null hypothesis of a unit root is rejected in an ADF test, as seen in Table 5.

## 4. Results

In this section we present the results of the estimations of TVPs in the BCB's Taylor rule. The first subsection provides the results for the estimation of the Taylor rule used to gather the initial parameters for the state-space estimation of the rule. The second subsection shows the estimation that led to the standard errors $\upsilon^*_{1t}$ and $\upsilon^*_{2t}$ used to relate the endogenous regressors with the selected instrumental variables, allowing the adjustment of the state-space model in accordance with the method derived by Kim and Nelson. The third subsection shows the results of the Taylor-rule when the inflation parameter ($\beta$) varies over time. For this scenario, we model three types of variations of the $\beta$ parameter: the first when only $\beta$ is allowed to vary over time, the second when the output gap parameter $\gamma$ is allowed to vary alongside $\beta$, also assuming a random walk process, and finally, an estimation where only $\beta$ varies over time in the space-state model, but when the neutral real interest rate $\overline{rr}$ is fixed, and not estimated, assuming it is the arithmetic average of the real interest rate over the sample period. The fourth subsection shows the results of the BCB's Taylor rule when the inflation target can vary with time, the so-called implicit inflation target, and how it compares to the target explicitly set by the CMN under Brazil's IT framework.

### 4.1 OLS Estimation of Taylor Rule Parameters

The OLS estimation is used to set the initial parameters for the Kalman filter estimation. We add time dummies are added to mitigate residuals and outliers at specific dates, ensuring a white-noise process. These include dummies for Q4 2003, during the first year of the Luiz Inacio Lula da Silva administration, Q1 2009, given the Great Financial Crisis, Q3 2011 and Q2 2012, given outlier data points and Q3 2018 due to the 2018 trucker strike.

Most of the estimated parameters, seen in autoreft:a4-01, are significant at the five-percent level. The smoothing coefficient variable ($\rho$) of 0.94 reinforces a notion of significant interest rate inertia. The value is in line with other Taylor rule estimations, like Modenesi (2011), which found a 0.92 value for the autoregressive term, rein, and Campos (2015), which had a coefficient of 0.964. The neutral real interest rate ($\overline{rr}$) of 0.053, or 5.3 % also appears to be a reasonable estimation, considering the period of higher real interest rates in Brazil in the early and mid-2000s, and appears to be within the median range of time-varying estimations for the period, like the Laubach and Williams approach by Fonseca and Muinhos (2018), and Perrelli and Roache (2014). The first main variable which will be time-varying in the state-space model, ($\beta$) is significantly higher than one, at 3.84, hence not only following the Taylor principle, but pointing to a hawkish BCB over the sample period. The coefficient weight for output gap ($\gamma$) is also above one, at 2.37, but lower than the coefficient for the inflation parameter. The dummy coefficients are all statistically significant.

## 4.2 Estimating standard errors for endogenous regressors

In line with the Kim and Nelson (2006) two-step bias-correction method, the residuals for equations (11) and (12) are estimated using the instrumental variables described in section 3 and shown in figures 8 and 9 in Appendix B. The residuals series of the two regressions are used as variables $\nu_1$ and $\nu_2$, respectively, in the Taylor rule estimations.

**Table 1.** OLS estimation of initial Taylor rule parameters

| Coefficient | Estimated Value | Standard Error | t-statistic | p-value |
|---|---|---|---|---|
| $\rho$ | 0.943704 | 0.025789 | 36.59342 | 0 |
| $rr$ | 0.05334 | 0.01847 | 2.887984 | 0.0054 |
| $\beta$ | 3.847894 | 2.035896 | 1.890025 | 0.0636 |
| $\gamma$ | 2.374615 | 1.052556 | 2.256047 | 0.0277 |
| $D_{2003Q4}$ | −0.021782 | 0.006213 | −3.505984 | 0.0009 |
| $D_{GFC(2009Q1)}$ | −0.017385 | 0.006004 | −2.895674 | 0.0053 |
| $D_{2011Q3}$ | −0.013989 | 0.005958 | −2.348115 | 0.0222 |
| $D_{2012Q2}$ | −0.015182 | 0.005987 | −2.535733 | 0.0138 |
| $D_{\text{Trucker Strike (Q3 2018)}}$ | 0.016710 | 0.006044 | 2.764695 | 0.0076 |
| $R^2 = 0.968325$ | | Adjusted $R^2 = 0.964101$ | | |

### 4.3 Estimating the time-varying inflation β parameter

Using the parameters from the OLS estimation as initial values, and the $\nu_1$ and $\nu_2$ residual series estimated in section 4.2 the Taylor rule equation (6a) is estimated. The coefficient used for $\sigma_e$ is the standard deviation of the residuals derived from the OLS estimation in section 4.1. Three different estimations are made for $\beta$ in this section. The first is a Taylor rule where the only parameter varying over time is that of $\beta$. The second estimation allows both $\beta$ and the output gap parameter $\gamma$ vary over time, to see if the latter has any impact on the estimation of the former. Lastly, we estimate $\beta$, but in a Taylor rule where the real interest rate $\overline{rr}$ is fixed, instead of being estimated along with the other parameters.

The first estimation, in Figure 1, shows the coefficient for the inflation parameter $\beta$ was above the Taylor principle for most of the sample period, in line with the OLS estimation in section 4.1, but dropped significantly starting in 2010, with the coefficient below one between Q1 2012 and Q4 2017, with the coefficient being negative between Q4 2012 and Q3 2015, indicating a dovish shift in the BCB's monetary policy preferences in the period. The coefficient recovers starting in 2016.

The second estimation, where the output gap parameter is also allowed to vary over time, shows similar results, as seen in Figure 2, indicating a hawkish response to inflation by the BCB in the first years of the sample, but a decline starting in Q2 2011. The coefficient falls below the Taylor principle of one in Q4 2012, like in the first estimation, but once the output gap is included, the coefficient never climbs back above one in the series.

Meanwhile, while not the main point of analysis in this study, it is worth also looking at the behavior of the output gap coefficient over time (Figure 3). The coefficient is significantly lower than the one estimated through OLS in section 4.1, being negative for a part of the sample but increasing for part of the period where the BCB became more dovish between late 2011 and 2014. The estimation also points to an increase in the output gap coefficient starting in 2017, becoming slightly above the 2013 levels in Q1 2020.

Finally, a third estimation is made, though this time, like in our first estimation, only $\beta$ is allowed to vary over time, but we use a fixed value for the natural real rate coefficient $\overline{rr}$, instead of estimating it. For this estimation, seen in Figure 4, the mean of the real interest rate over the sample period, 5.97 %, is used. The use of the fixed coefficient for $\overline{rr}$ is used in Leigh and Klein.

Overall, the three results of $\beta$ (Figure 5) point to similar trends over time. In all three cases there is a significant decline in the weight of inflation in the BCB's Taylor rule around 2011, consistent with criticism of an overly stimulative monetary policy by then-governor Alexandre Tombini, though with some recovery in the value of the parameter around 2016.

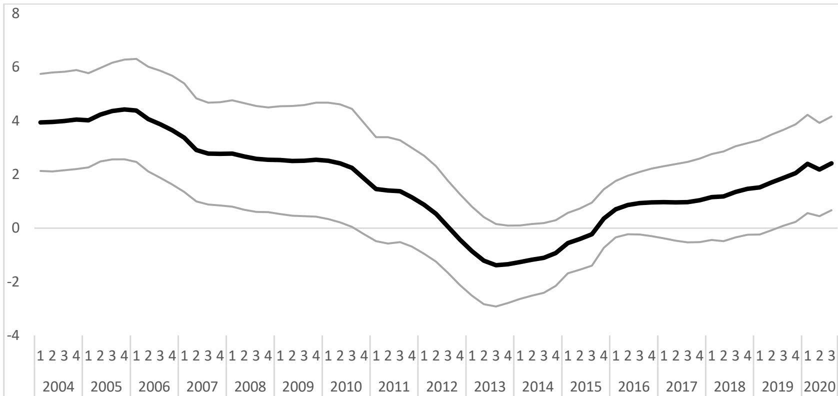


**Figure 1.** Model 1 – Estimating $\beta$ as only time-varying parameter (plus/minus two standard errors)

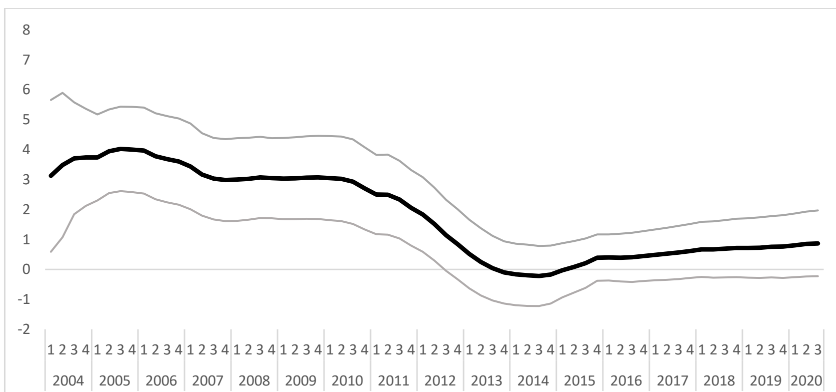


**Figure 2.** Model 2 – Estimating $\beta$ and $\gamma$ as time-varying parameters: $\beta$ plus/minus two standard errors

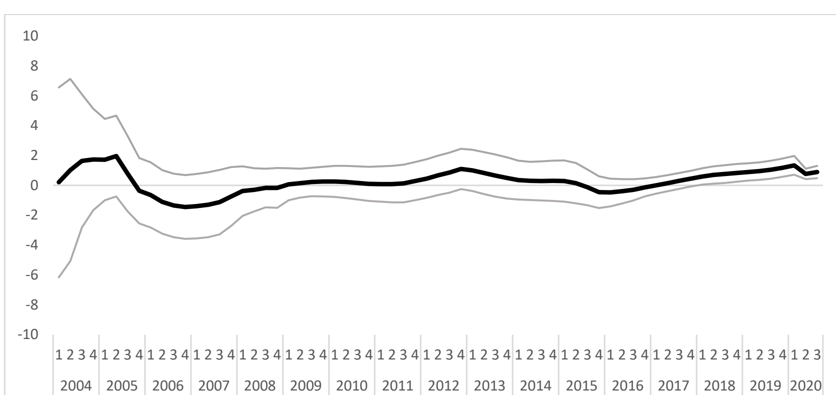


**Figure 3.** Model 2 – Estimating $\beta$ and $\gamma$ as time-varying parameter: $\gamma$ plus/minus two standard errors

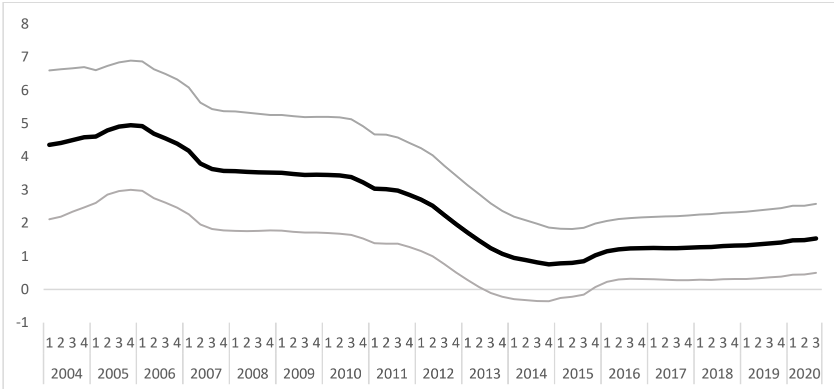


**Figure 4.** Model 3 – Estimating $\beta$ as time-varying parameter with fixed $\overline{rr}$: $\beta$ plus/minus two standard errors

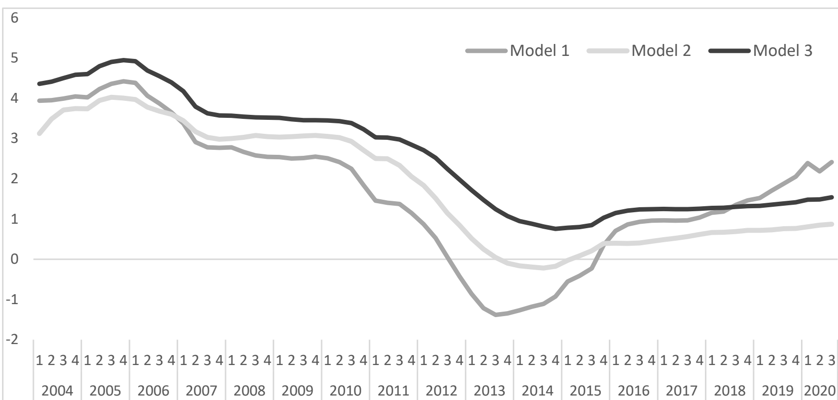


**Figure 5.** Comparison of models 1–3

## 4.4 Estimating the implicit inflation target of the BCB

The second aim of this paper is to estimate the implicit inflation target of the BCB through the TVP model. The results show significant variation of the implicit inflation target set by the BCB over time. Although most of the estimated values stay within the official inflation target bands, the implicit inflation target has, at times, left the established bands—despite staying within these bands if the two standard errors of the band are also accounted for.

Figure 6 compares the estimated implicit inflation target to the official target set by the CMN and its bands. With the time series comparison, we can visually observe five distinct eras, or regimes, in the BCB's decisionmaking.

First, comes the 2004–2008 when the BCB's implicit target was below the center of the CMN target, though within the set band. The more hawkish approach

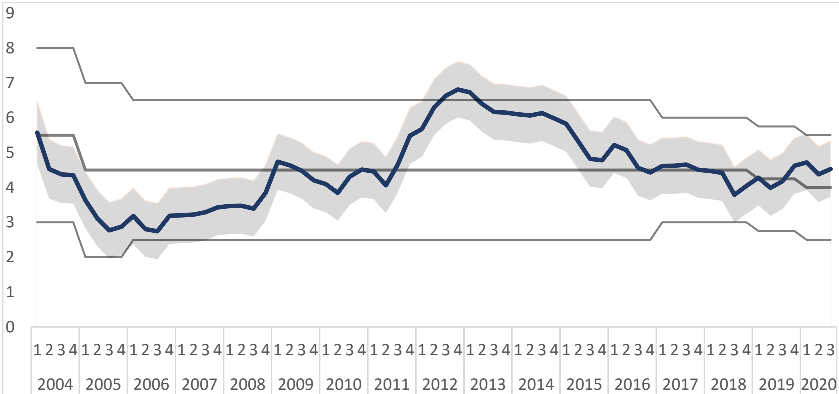


**Figure 6.** Implicit inflation target and official inflation target (plus/minus two standard errors, %)

in the estimation is in line with the inflation parameter coefficients estimated in section 4.3, which reach levels of three, or even above four in a few quarters. These years correlate with the first, and beginning of the second Lula terms, with the BCB still headed by Meirelles.

Second comes the period between Q3 2008 and Q2 2011, when the implicit inflation target estimated largely aligns with the center of the CMN inflation target. The data shows a rapid increase in the implicit inflation target around the Great Financial Crisis, with the BCB taking on a more accommodative tone, though still in line with the CMN target, ultimately being less hawkish than in the previous years. This estimation is also largely in line with what was seen in section 4.3, with the inflation coefficient falling from the three-to-four range, to a rather stable value around three between 2008 and 2011; still above the Taylor principle coefficient of one.

Third comes the period between Q3 2011 and Q2 2015, when the estimated implicit inflation target for the BCB was significantly above the center of the CMN inflation target. Between Q2 2012 and Q2 2013, the estimated implicit target values are not only above the center of the target, but also above the upper band set by the CMN. This result occurs during the Tombini years at the BCB where the bank followed a much more stimulative monetary policy. The implicit inflation target is also in line with the significant drop in the inflation parameters seen in section 4.3, when the coefficient for inflation hovered around zero.

Fourth is the period between Q3 2015 and Q3 2019, when the BCB's implicit target follows the explicit target. The years followed the 2015–2016 economic downturn and the impeachment of President Dilma Rousseff, which also included a change at the BCB, with Ilan Goldfajn being appointed by President Michel Temer to replace Tombini in the new administration. The period saw a fall in inflationary pressures and interest rates as well. The inflation coefficients estimated in section 4.3 appear to have returned above the Taylor principle.

Lastly, starting in Q4 2019 and through to the end of the sample at Q3 2020, the BCB appears to have taken a more dovish approach to monetary policy. To be sure, it is difficult to conclude that the BCB's policy choices during Q4 2019 were significantly different than those the previous quarters, but the more stimulative approach becomes clearer in the first two quarters of 2020 when the BCB responded to the shock in output caused by the Covid-19 pandemic.

We can also assess how the implicit target deviations from the official target varies over time, and how inflation expectations vary during the period.

Visually assessing Figure 7, it can be noted the time series of the implicit target deviation from the official target time series appears to "precede" changes to inflation expectations as surveyed by the BCB. As such, it is worth testing whether this relationship is Granger causal, Hence, a Granger causality test with four lags is run to see whether the implicit inflation target can help predict inflation expectations. Results are displayed in Table 2.

The F-Statistic in the Granger causality test is significant enough to reject the null hypothesis that the implicit target's deviation from the official target does not Granger cause inflation expectations, almost to the 1 % level. Hence, there could

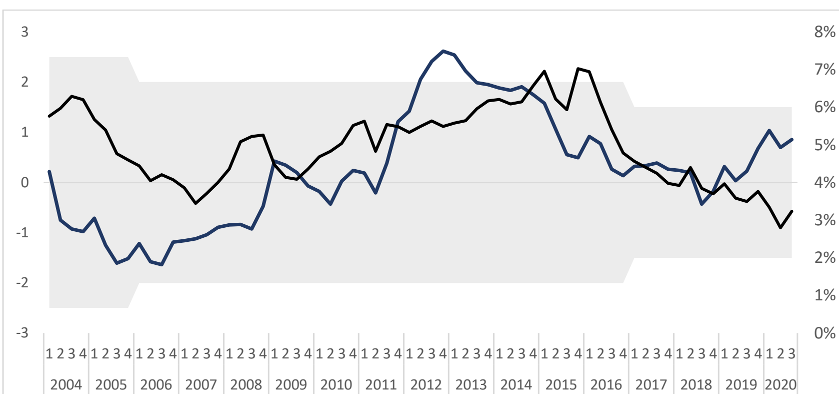


**Figure 7.** Implicit inflation target deviation from official target (percentage points), and 12-month expected inflation rate (RHS, %)

**Table 2.** Granger causality test between the implicit target deviation from the official target, and inflation expectations

| | F-Statistic | p-value |
|---|---|---|
| Implicit target deviation from official target *does not* Granger cause inflation expectations | 3.26156 | 0.0177 |

be an argument to be made that larger deviations between the implicit and official inflation targets in the BCB's Taylor rule can help predict dislocations in inflation expectations. To be sure, the above is a simple test, and further research should be done to deepen this understanding.

Lastly, the Table 3 breaks down the implicit inflation target for the sample period under each governor's tenure, showing descriptive statistics for each. The mean squared deviation of each respective governor's estimated implicit inflation target, from the explicit center of the CMN target is also measured. With this metric, it can be noted that the Goldfajn tenure had the lowest mean squared deviation from the explicit target, using the implicit target estimated above.

**Table 3.** Descriptive statistics of implicit targets by BCB governor

| | Meirelles | Tombini | Goldfajn | Campos Neto |
|---|---|---|---|---|
| Number of quarters in sample period ($n$) | 30 | 22 | 10 | 7 |
| Below bottom band | 0<br>(0%) | 0<br>(0%) | 0<br>(0%) | 0<br>(0%) |
| Between center and bottom band | 24<br>(80%) | 2<br>(9%) | 5<br>(50%) | 2<br>(29%) |
| Between center and upper band | 6<br>(20%) | 17<br>(77%) | 5<br>(50%) | 5<br>(71%) |
| Above upper band | 0<br>(0%) | 3<br>(14%) | 0<br>(0%) | 0<br>(0%) |
| Mean squared deviation from official inflation target | 1.168262 | 1.957557 | 0.078214 | 0.16439 |

## 5. Conclusion

Our main findings are that the BCB's approach to responding to inflation has, since 2003, become more dovish. Whereas early in the sample period, the bank took a much harsher response to inflationary pressures, since 2011 monetary policy has become much more stimulative, and though for a time the estimated time-varying coefficient for a forward-looking Taylor rule was below one, the response to inflationary pressures has strengthened since 2016, though not to the levels seen before the 2011 dovish turn in the bank's monetary preferences. The second conclusion of the study, which is inherently related to the first, is that the BCB's implicit inflation target has stayed largely within the bands set by the CMN, though the deviations between the estimated implicit target and the actual target vary over time, notably leading to implicit inflation targets that were outside the CMN's band.

The results are also largely in line with other research for the case of the BCB for the case of the inflation parameter varying across time: the latest observations used in Policano overlap with this study, and despite the different specifications, show the coefficient for the inflation gap from its target is around 3–3.5 in 2003–2004, but begins dropping in 2004, somewhat like our conclusions. Aragon and Medeiros, in their TVP estimation of a Taylor rule for Brazil, though with a different sample and slightly different specification, find the BCB's response coefficient to inflation dropped below one after 2010. Additionally, Rodrigues estimated the BCB's reaction function using a Markov switching estimation, finding that the majority of the time the BCB reacted in great part to the inflation gap to its target, though at times certain estimated regimes pointed to a more stimulative monetary policy, such as between 2011 and 2012.

The conclusion also shows the BCB has faced multiple changes in its implicit inflation target, which have varied across presidential and BCB terms. Visually, and unsurprisingly, the implicit inflation target's deviation from the official target also shows some relation with inflation expectations over time, as seen on Figure 7. The results hence show much less commitment to the official inflation target in certain times, namely between 2011 and 2016, though most of the data sample still shows the BCB has been responsible when it comes to the inflation target, as the estimates for the implicit target remain largely within the bands allowed by the CMN.

A point to note is the Granger causality between the deviation of the implicit target from the official target and inflation expectations. While what was done in this paper is limited, further study into the relationship could yield relevant conclusions to understanding the impact of the BCB's monetary policy preferences on economic agents' expectations.

Ultimately, the main contribution of this study is to provide another framework to assess the BCB's decisions over time, providing such in an updated TVP framework for, as well as present an implicit inflation target framework to easily provide a visual representation of the BCB's decisions and the official target over time.

Lastly, it should be noted this study can still be advanced, with different specifications of monetary rules, such as the inclusion of an exchange rate variable to the Taylor rule, or further adjustments to the bias-correction method used. This research should be followed-up over time for a continuous assessing of changes on the TVPs in the Taylor rule.

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## Appendix A. Kim and Nelson Methodology

The Kim and Nelson two-step method sets $J = 1$, also decomposing the inflation gap and output gap into two sections: the first pertaining to predicted components and the second pertaining to prediction errors, in the following matrix structure:

$$\begin{bmatrix}\pi_{t,1}\\ g_{t,1}\end{bmatrix} = E\left[\begin{matrix}\pi_{t,1}\\ g_{t,1}\end{matrix}\middle|\psi_{t-1}\right] + \begin{bmatrix}\upsilon_{1,t|t-1}\\ \upsilon_{2,t|t-1}\end{bmatrix}, \tag{18}$$

$$\begin{bmatrix}\upsilon_{1,t|t-1}\\ \upsilon_{2,t|t-1}\end{bmatrix} = \Omega_{t|t-1}^{1/2}\begin{bmatrix}\upsilon_{1,t}^{*}\\ \upsilon_{2,t}^{*}\end{bmatrix}, \qquad \begin{bmatrix}\upsilon_{1,t}^{*}\\ \upsilon_{2,t}^{*}\end{bmatrix} \sim i.i.d.\,\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\right), \tag{19}$$

where $\psi_{t-1}$ is the information set up to period $t-1$. Furthermore, $\Omega_{t|t-1}$ is the conditional variance covariance matrix, for a vector of prediction errors

$$\begin{bmatrix}\upsilon_{1,t|t-1}\\ \upsilon_{2,t|t-1}\end{bmatrix}$$

which is time-varying and obtained from (13) and (14).

Continuing with Kim and Nelson's derivation for the model, a $2\times 1$ vector of standardized prediction errors, $\upsilon_t^{*} = \begin{bmatrix}\upsilon_{1,t}^{*} & \upsilon_{2,t}^{*}\end{bmatrix}'$, in which the covariance between the standardized predictions and the error term in the signal equation is

$$\begin{bmatrix}\upsilon_t^{*}\\ e_t\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}I_2 & \rho\sigma_{e,t}\\ \rho'\sigma_{e,t} & \sigma_{e,t}^2\end{bmatrix}\right), \tag{20}$$

where $\rho = \begin{bmatrix}\rho_1 & \rho_2\end{bmatrix}'$ is a correlation vector. Kim and Nelson follow the Cholesky decomposition of the above covariance matrix:

$$\begin{bmatrix}\upsilon_t^{*}\\ e_t\end{bmatrix} = \begin{bmatrix}I_2 & 0_2\\ \rho'\sigma_{e,t} & \sqrt{(1-\rho'\rho)}\sigma_{e,t}\end{bmatrix}\begin{bmatrix}\epsilon_t\\ \omega_t\end{bmatrix},$$

$$\begin{bmatrix}\epsilon_t\\ \omega_t\end{bmatrix} \sim i.i.d.\,\mathcal{N}\left(\begin{bmatrix}0_2\\0\end{bmatrix}, \begin{bmatrix}I_2 & 0_2\\ 0_2' & 1\end{bmatrix}\right), \tag{21}$$

where $0_2$ is a vector of zeroes.

# Appendix B. Estimation Output for TVP models

**Table 4.** Unit root Test statistics

| Variable | ADF | PP | KPSS |
|---|---|---|---|
| Real interest rate ($rr_t$) | -0.3151 | -2.6802 | 0.1206* |
| Inflation expectations ($E[\pi]$) | -4.6619*** | -4.4178*** | 0.1473** |
| Inflation target ($\pi^*$) | -3.9597** | -4.0871** | 0.0544 |
| Output gap ($\gamma$) | -1.7250 | -3.5214** | 0.2153** |
| US Dollar to Brazilian Reais ($dlfx$) | -6.3676*** | -6.1147*** | 0.0492 |
| WTI oil prices ($dloil$) | -7.2571*** | -7.3557*** | 0.0585 |
| Monthly industrial production ($dlpim$) | -8.7574*** | -8.6235*** | 0.0751 |

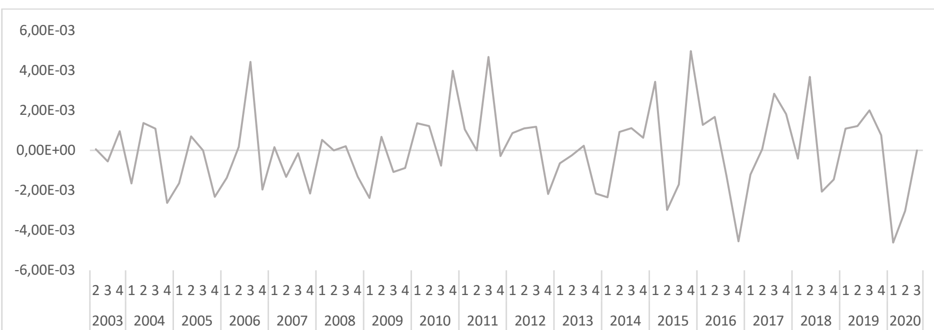


**Figure 8.** Residuals of instrumental variable estimation for $E[\pi_t]$

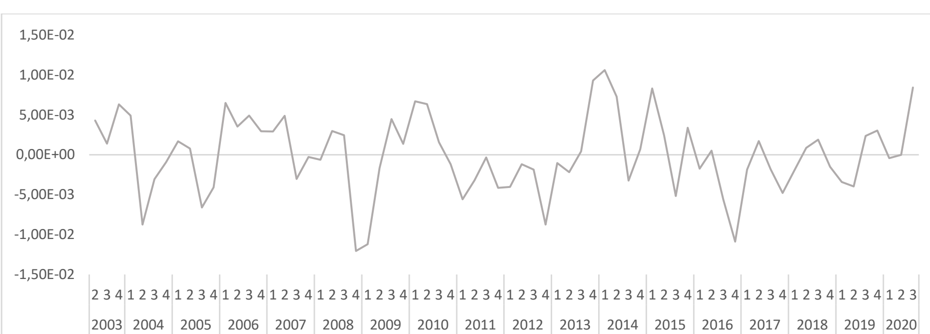


**Figure 9.** Residuals of instrumental variable estimation for $\gamma_t$

**Table 5.** Unit root with breakpoint test statistics

| Variable | ADF |
|---|---|
| Real interest rate ($rr_t$) | -5.7029** |
| Output gap ($\gamma$) | -5.8752*** |

*** p < 0.01 ** p < 0.05 * p < 0.10

**Table 6.** Model 1 – Estimating $\beta$ as only time-varying parameter

| Coefficient | Estimated Value | Standard Error | z-Statistic | p-value |
|---|---|---|---|---|
| $\rho$ | -4.080052 | 0.250062 | -16.31618 | 0.0000 |
| $\overline{rr}$ | 0.069156 | 0.005964 | 11.59481 | 0.0000 |
| $\gamma$ | 0.520082 | 0.217832 | 2.387535 | 0.0170 |
| $v_1$ | -670.1163 | 2026.426 | -0.330689 | 0.7409 |
| $v_2$ | -702.9792 | 1045.290 | -0.672521 | 0.5013 |
| $TVP$ | Final State | Root MSE | z-Statistic | p-value |
| $\beta$ | 2.412928 | 0.905056 | 2.666053 | 0.0077 |
| Log-likelihood | 38.36762 | Akaine I.C. | | -0.967177 |
| Parameters | 5 | Schwarz I.C. | | -0.805286 |
| Diffuse priors | 1 | Hannan-Quinn I.C. | | -0.902949 |

**Table 7.** Model 2 – Estimating $\beta$ and $\gamma$ as time-varying parameter

| Coefficient | Estimated Value | Standard Error | z-Statistic | p-value |
|---|---|---|---|---|
| $\rho$ | 0.000528 | 0.000112 | 4.699920 | 0.0000 |
| $\overline{rr}$ | 0.067951 | 0.003477 | 19.54500 | 0.0000 |
| $v_1$ | -262.2820 | 337.8877 | -0.776240 | 0.4376 |
| $v_2$ | -33.63888 | 156.2058 | -0.215350 | 0.8295 |
| $TVP$ | Final State | Root MSE | z-Statistic | p-value |
| $\beta$ | 0.870547 | 0.558089 | 1.559870 | 0.1188 |
| $\gamma$ | 0.898746 | 0.256183 | 3.508214 | 0.0005 |
| Log-likelihood | 139.3330 | Akaine I.C. | | -3.922694 |
| Parameters | 4 | Schwarz I.C. | | -3.793181 |
| Diffuse priors | 2 | Hannan-Quinn I.C. | | -3.871312 |

**Table 8.** Model 3 – Estimating $\beta$ as time-varying parameter with fixed $\overline{rr}$

| Coefficient | Estimated Value | Standard Error | z-Statistic | p-value |
|---|---|---|---|---|
| $\rho$ | -3.710086 | 0.212645 | -17.44736 | 0.0000 |
| $\gamma$ | 0.398050 | 0.305797 | 1.301681 | 0.1930 |
| $v_1$ | -954.5989 | 2589.041 | -0.368708 | 0.7123 |
| $v_2$ | -657.2078 | 1216.875 | -0.540078 | 0.5891 |
| $TVP$ | Final State | Root MSE | z-Statistic | p-value |
| $\beta$ | 1.538786 | 0.523789 | 2.937801 | 0.0033 |
| Log-likelihood | 32.34217 | Akaine I.C. | | -0.821512 |
| Parameters | 4 | Schwarz I.C. | | -0.691999 |
| Diffuse priors | 1 | Hannan-Quinn I.C. | | -0.770130 |

**Table 9.** Model 4 – Implicit inflation target

| Coefficient | Estimated Value | Standard Error | z-Statistic | p-value |
|---|---|---|---|---|
| $\rho$ | 0.742983 | 0.053893 | 13.78630 | 0.0000 |
| $\overline{rr}$ | 0.051069 | 54259861 | 9.41E-10 | 1.0000 |
| $\beta$ | 3.622114 | 0.524541 | 6.905302 | 0.0000 |
| $\gamma$ | 0.540605 | 0.278322 | 1.942370 | 0.0521 |
| $v_1$ | -239.9813 | 71.22085 | -3.369537 | 0.0008 |
| $v_2$ | -35.39408 | 25.51706 | -1.387075 | 0.1654 |
| $TVP$ | Final State | Root MSE | z-Statistic | p-value |
| $\pi^*$ | 0.045285 | 0.004865 | 9.309075 | 0.0000 |
| Log-likelihood | 228.8570 | Akaine I.C. | | -6.459622 |
| Parameters | 6 | Schwarz I.C. | | -6.265352 |
| Diffuse priors | 1 | Hannan-Quinn I.C. | | -6.382549 |