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Adjustment of mixed nonlinear models on Blackberry fruit growth

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Abstract: Blackberry fruits belong to the *genus Rubus,* are fruits more cultivated in temperate climate in the summer, with low luminosity and low temperature in the winter. These fruits have as characteristic the quickperishingafter harvest and regression models, more specifically, nonlinear models, single or double sigmoid growth curve, are more recommended to model the growth of living beings. Several authors have used these models, considering only the average data of individuals under study; however, they do not consider the variability between them. One way to better capture the variability between individuals is by using mixed-effects nonlinear models that, by definition, combine the fixed and random part in the same model. Data used in this work were diameter and length of 'Choctaw' blackberry fruits, . The random effects of models were tested on parameters, with some steps, in order to reach the most appropriate model. For fixed-effects models, the least squares method was used, and for mixed models, the restricted likelihood was used. To reach the model that best fits data, the fit quality criteria (R^2, AIC_c and TRV) were used. For fruit diameter, the simple sigmoid nonlinear model was the logistic with random effect in β_1 and β_2, and for fruit length, the model was the Logistic + Logistic, with random effect in β_1 and β_4.

Index terms: 'Choctaw' cultivar; Restricted likelihood; Variability; Estimation.

Ajuste de modelos não lineares mistos no crescimento dos frutos de amora-preta

Resumo: Os frutos de amora-preta pertencem ao gênero *Rubus*, são frutos mais cultivados em clima moderado, no verão, com pouca luminosidade e baixa temperatura no inverno. Esses frutos têm por característica perecerem de forma rápida após a colheita, e os modelos de regressão, mais especificamente, os não lineares, de crescimento sigmoide simples ou duplo, são mais recomendados para se modelar o crescimento de seres vivos. Diversos autores utilizam esses modelos, levando em consideração apenas os dados médios dos indivíduos em estudo; contudo, não levam em consideração a variabilidade existente entre eles. Uma forma de captar melhor a variabilidade existente entre os indivíduos é utilizando os modelos não

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lineares de efeitos mistos que, por definição, combinam a parte fixa e aleatória no mesmo modelo. Os dados utilizados, neste trabalho, foram o diâmetro e o comprimento dos frutos de amora-preta, cultivar Choctaw. Os efeitos aleatórios dos modelos foram testados nos parâmetros, com algumas etapas, a fim de chegar ao modelo mais adequado. Para os modelos de efeitos fixos, o método de mínimos quadrados foram utilizados, e para os modelos mistos a verossimilhança restrita. Para se chegar ao modelo que melhor se ajuste aos dados, os critérios qualidade de ajuste (R^2_{aj}, AIC_c and TRV) foram utilizados. Para o diâmetro dos frutos, o modelo não linear sigmoide simples foi o logístico, com efeito aleatório no β_1 and β_2; já, para o comprimento, foi o Logístico + Logístico, tendo o efeito aleatório no β_1 and β_4.

Termos de indexação: Cultivar Choctaw; Verossimilhança restrita; Variabilidade; Estimação.

Introduction

Small fruits, such as blackberries, strawberries, raspberries, gooseberries, blueberries, among others, have been growing in importance, quality and volume. The cultivation of these species is characterized by the low cost of implantation, production and good economic return. The fresh consumption of fruits maintains their physicochemical, nutritional and biological properties. However, most of the production is destined to the production of processed products such as jellies, juices, pulp for ice cream, among others. The consumption in the processed form is justified by the fact that these fruits have quick perishing, that is, after harvesting, their shelf life is short (TADEU et al, 2015).

Blackberry belongs to the genus *Rubus*, is a small fruit, with good adaptation to temperate climate. Its cultivation for marketing began in Europe in the 16thcentury, later taken to the USA and arriving in Brazil in the 1970 (RASEIRA; FRANZON, 2012). This is a crop with quick financial return for the producer, but little explored in Brazil, with plantations occurring in the states of Rio Grande do Sul, Espírito Santo, Rio de Janeiro and southern Minas Gerais (FACHINELLO et al., 2011).

These fruits are subjectively harvested by color and size (CAVALINI et al., 2006). The conservation time of fresh fruits is short, so identifying the optimal harvest time is very important, increasing the consumption time of fresh fruits. With the rapid loss of post-harvest quality, there is a very large limitation for consumption in the fresh form; therefore, it is very important to use techniques to mitigate losses.

Thus, choosing the ideal harvest time is very important and knowing the growth and development curve of this fruit can be useful in this process, helping to reduce the rapid loss of quality after the harvest period, which is the main problem for fruits intended for fresh consumption.

Linear or non-linear regression models can be of great value in obtaining statistical models to objectively describe how this growth pattern occurs, highlighting non-linear regression models that, in the case of simple sigmoid, have less parameters and with biological interpretation (MAZZINI et al., 2003).

However, in several studies with longitudinal data, such as those carried out by Muianga et al., (2016), Ribeiro et al., (2018a), Silva et al., (2020), Miranda et al. (2021) and Silva et al., (2021b), researchers do not take into account the variability among researched fruits. Among the various ways to verify the variability among fruits, modeling via mixed models can be highlighted (WYZYKOWSKI et al., 2015; SARI et al., 2018; SARI et al., 2019b). Mixed models are the result of a technique where the fixed part and the random part are combined (PINHEIRO; BATES, 2004), and the advantage of modeling in this way is being able to capture the variability among fruits under study, with the inclusion of random effect on one or more parameters. SARI et al. (2018) studied the random effect of growth, verifying the inclusion of random effects to the parameters of nonlinear growth models and concluded that the inclusion of random effects is efficient.

As in fixed-effects models, for the modeling of random effects, it is necessary to assign initial values for the method to interactively calculate the estimates up to a stopping criterion and, as in fixed models, it is not an easy task. Estimation can be performed by some methods, the restricted likelihood being the most used (LINDSTROM; BATES, 1990). Restricted likelihood is a generalization of likelihood, but this method takes into account the fixed part and the random part in the model (PINHEIRO; BATES, 2004).

There are advantages in mixed models in relation to fixed-effect ones, that in addition to verifying the variation among individuals, these models are more flexible in relation to assumptions, incorporating dependence and heteroscedasticity to the model via the variance-covariance matrix (PINHEIRO; BATES, 2004).

In literature, most studies on fruit growth are carried out with simple sigmoid models; however, in some cases, this growth occurs in two development stages. In the first stage, growth is slow at the beginning, changing to fast and stabilizing at the end, and in a second stage, the same characteristics are found, becoming a double sigmoid model (FERNANDES et al, 2017; SILVA et al., 2020, FERNANDES et al., 2022).

Thus, the aim of the present study was to verify the growth pattern through non-linear sigmoid single and double regression models, adding, if necessary, one or more parameters of random effects, applying this methodology to data obtained from 'Choctaw' blackberry in relation to fruit diameter and length.

Material and methods

Data were extracted from (TADEU et al., 2015) and represent the results of the experiment carried out at the Federal University of Lavras, Lavras, MG, from January 2012 to January 2014. The climate in the region is Cwb type (mesothermal or high-altitude tropical climate), with dry winter and rainy summer, according to the Koppen classification. The variables under study were diameter and length of 'Choctaw' blackberries. Seedlings were planted in 2009 with "T" vertical cordon with 60 cm of distance and 80 cm of height and 3.0 m x 0.5 m spacing.

Tadeu et al. (2015) carried out the initial study aiming to compare the effect of conventional (control) and drastic summer pruning. From this study, diameter and length measurements of the 'Choctaw' cultivar were taken, in mm. Such measurements were collected with the aid of digital caliper (model King Tools 150 mm, Cia, São Paulo, SP). The choice of fruits took into consideration the development stage, that is, fruits whose petals and anthers already fell, at the same time, were chosen considering longitudinal data, that is, the same fruits had measurements verified until harvest. Twelve measurements were collected over time, the first being three days after an thesis (DAA) and the last after thirty-five DAA.

The parameterization used in Table 1 for model adjustments was taken from Fernandes et al. (2015), as follows. In this parameterization, β_1 and β_4 represent the maximum expected growth in each growth curve, β_2 and β_5 the inflection point in the respective curves, except for the Brody model, which does not have an inflection point, β_3 and β_6 the parameters linked to the growth rate in each curve, representing the fixed part and b_1, b_2, b_3, b_4, b_5, b_6 are the random effects parameters associated with each of the fixed-effects parameters, with b_1 and b_2 associated with β_1 and β_4, b_2 and b_5 associated with β_2 and β_5 and b_3

and b_6 associated with β_3 and β_6, respectively. With b_i the random effects of the model independent and identically distributed, $b_i \sim N(0, \sigma^2 \boldsymbol{D})$, $\sigma^2 \boldsymbol{D}$, the matrix of variances and covariances and ϵ_{ij} the random error associated with the model, with ϵ_{ij}.

Table 1 - Expressions of Logistic, Gompertz, Logistic + Logistic, Gompertz + Gompertz, Logistic + Brody and Gompertz + Brody adjusted models of fixed and random effects to describe the diameter and height growth of 'Choctaw' blackberry.

Model	Random effect	Expression
Logistic	-------	$\frac{\beta_1}{1+e^{\beta_3(\beta_2-x_{ij})}}+\epsilon_{ij}$
	β_1	$\frac{(\beta_1+b_1)}{1+e^{\beta_3(\beta_2-x_{ij})}}+\epsilon_{ij}$
	β_3	$\frac{\beta_1}{1+e^{(\beta_3+b_3)(\beta_2-x_{ij})}}+\epsilon_{ij}$
	$\beta_1\beta_3$	$\frac{(\beta_1+b_1)}{1+e^{(\beta_3+b_3)(\beta_1-x_{ij})}}+\epsilon_{ij}$
	$\beta_1\beta_2$	$\frac{(\beta_1+b_1)}{1+e^{\beta_3((\beta_2+b_2)-x_{ij})}}+\epsilon_{ij}$
Gompertz	-------	$\beta_1 e^{-e^{\beta_3(\beta_2-x_{ij})}}+\epsilon_{ij}$
	β_1	$(\beta_1+b_1)e^{-e^{\beta_3(\beta_2-x_{ij})}}+\epsilon_{ij}$
	β_3	$\beta_1 e^{-e^{(\beta_3+b_3)(\beta_2-x_{ij})}}+\epsilon_{ij}$
	$\beta_1\beta_2$	$(\beta_1+b_1)e^{-e^{\left(\beta_3\left((\beta_2+b_2)-x_{ij}\right)\right)}}$
Brody +Logistic	--------	$\beta_1\left(1-e^{\beta_3(\beta_2-x)}\right)+\frac{(\beta_4-\beta_1)}{1+e^{\beta_5(\beta_6-x_{ij})}}+\epsilon_{ij}$
	$\beta_1\beta_4$	$(\beta_1+b_1)\left(1-e^{\beta_3(\beta_2-x_{ij})}\right)+\frac{((\beta_1+b_1)-(\beta_4+b_4))}{1+e^{\beta_5(\beta_6-x_{ij})}}+\epsilon_{ij}$
Logistic+ Logistic	--------	$\frac{\beta_1}{1+e^{\beta_3(\beta_2-x_{ij})}}+\frac{(\beta_4-\beta_1)}{1+e^{\beta_5(\beta_6-x_{ij})}}+\epsilon_{ij}$
	$\beta_1\beta_4$	$\frac{(\beta_1+b_1)}{1+e^{\beta_3(\beta_2-x_{ij})}}+\frac{((\beta_4+b_4)-(\beta_1+b_1))}{1+e^{\beta_5(\beta_6-x_{ij})}}+\epsilon_{ij}$

At first, in order to have prior knowledge of how these measures behave over time, scatter plots and boxplots were built with the ggplot2 package of the R software (WICHAM, 2016). Subsequently, fixed-effects nonlinear regression models were adjusted, without verifying the assumptions of residuals. Since these fixed-effects models were adjusted, the following steps were used to verify in which parameters the random effect will be included. Step 1 is a method taken from Pinheiro and Bates (2004), which consists of making individual confidence intervals (95%) in parameters illustrated in Figure 1. If all intervals overlap each other, it is an indication that there is no need to include the random effect in the parameter; however, if at least one does not overlap with the others, there is a need to include this effect in the parameter.

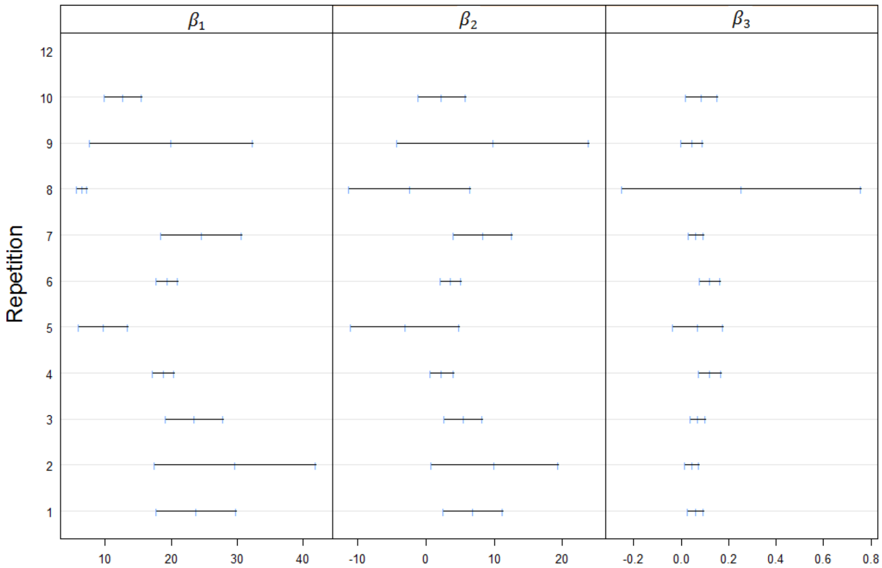

Figure 1- Graphic representation of the 95% confidence intervals for estimates of the logistic non-linear model parameters.

Step 2, readjust the models combining the inclusion of random effects to parameters using the comparison criteria between models ($R^2_{aj}, AIC_c, and\ TRV$), thus reaching a decision on which parameters need to be included (WYZYKOWSKI, et al. , 2014).

After choosing the two models to describe diameter and length, these models were adjusted once again, adding the structure of the variance-covariance matrix of the random effects to parameters. The matrices chosen were varPower and varExp, both implemented in the nlme package (PINHEIRO et al. (2021)) and the first-order autoregressive term (AR1).

To adjust the models, the R software (R DEVELOPMENT CORE TEAM, 2022) was used, estimating the parameters of fixed-effects models via ordinary least squares and for mixed models restricted maximum likelihood, using the nlme packages (PINHEIRO et al., 2021). The initial values for the iterative method to start the iteration was a graphic analysis with the nlsList and manipulate functions (Allaire, 2014; Diel et al., 2019).

To verify the homogeneity of variances and normality of residues, the Breusch-Pagan (BREUSCH; PAGAN, 1979) and Shapiro-Wilk (SHAPIRO; WILK, 1965) tests were used, respectively. To assess the existence of residual autocorrelation, the Durbin-Watson test was used (DURBIN; WATSON, 1951). In addition to hypothesis testing, graphical analysis was used to verify these assumptions. To verify the fit quality of models, the parametric non-linearity measure was used, and as mentioned by Sari et al. (2019a), values of parametric nonlinearity measures less than 1 indicate that the parameterization used is acceptable. Models were compared using the following criteria: i) Adjusted determination coefficient $\left(R^2_{aj}\right)$, obtained by: $R^2_{aj} = 1 - \left[\frac{(1-R^2)(n-1)}{n-p}\right]$, where R^2 is the determination coefficient, $\boldsymbol{n}$ is the number of times in which measurements were taken and $\boldsymbol{p}$ is the number of model parameters. ii) Corrected Akaike

Information Criterion (AKAIKE, 1974), given by: $AIC_c = AIC + \frac{2p(p+1)}{n-p-1}$, where $AIC = n\, ln\left(\frac{SQR}{n}\right) + 2p$, where is the sum of squares of residuals, $\boldsymbol{p}$ is the number of model parameters, $\boldsymbol{n}$ is the sample size and $\boldsymbol{ln}$ is the natural logarithmic operator, iii) Likelihood ratio test given by: $L = -2log(L_1/L_2)$. After choosing the models with the best fit quality values, the critical points were estimated for fruit diameter and length, according to Silva et al. (2021b). For all analyses, the significance level adopted was 5%.

Results and discussion

Initially, the exploratory analysis of data was carried out in order to verify the behavior of data over time. As observed in Table 2, the averages of the two characteristics studied at 35 DAA were 17.60 and 15.80 mm, receptively for diameter and length. These two averages can be used as the initial value of the asymptotic parameter of models (MISCHAN; PINHO, 2014; FRÜKAUF, 2022).

Table 2- Descriptive statistics (average and variance) for diameter and length of 'Choctaw' blackberry fruits, at each time, in mm.

Time	Average		Variance	
	Length	Diameter	Length	Diameter
1	5.52	4.34	0.24	0.31
5	7.38	6.87	1.22	1.82
8	7.71	8.62	4.57	4.68
10	9.73	9.71	4.52	11.50
13	10.40	11.10	6.43	13.00
16	11.50	11.70	8.14	14.90
20	11.60	11.70	12.10	16.40
22	12.70	12.50	12.50	18.70
24	13.10	13.30	14.80	18.90
28	14.30	14.00	24.50	28.60
31	16.00	15.20	29.60	33.70
35	17.60	15.80	34.60	46.60

Figures 2 and 3 show the scatter plots and boxplots of growth data (diameter and length) of individual fruits over time. It is possible to observe that, in both cases, at the beginning of the productive cycle, fruits have more homogeneous growth and, over time, there is greater growth variation (heterogeneous growth).

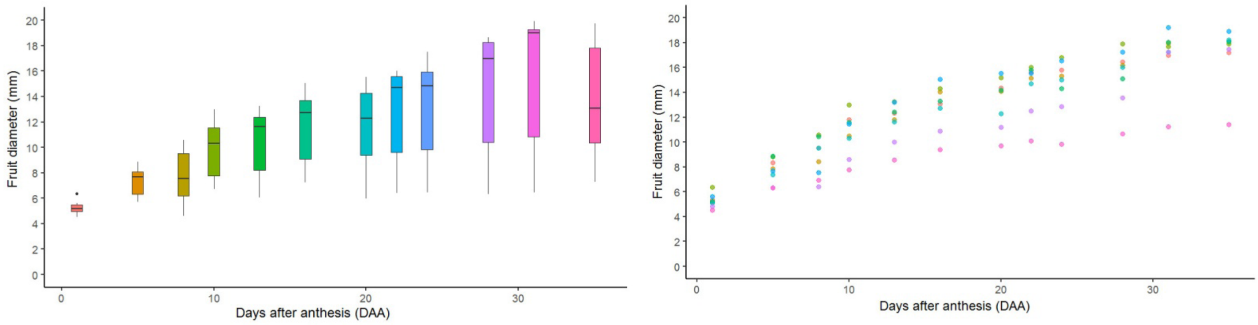

Figure 2 - Scatter plot and boxplot of 'Choctaw' blackberry diameter data over time.

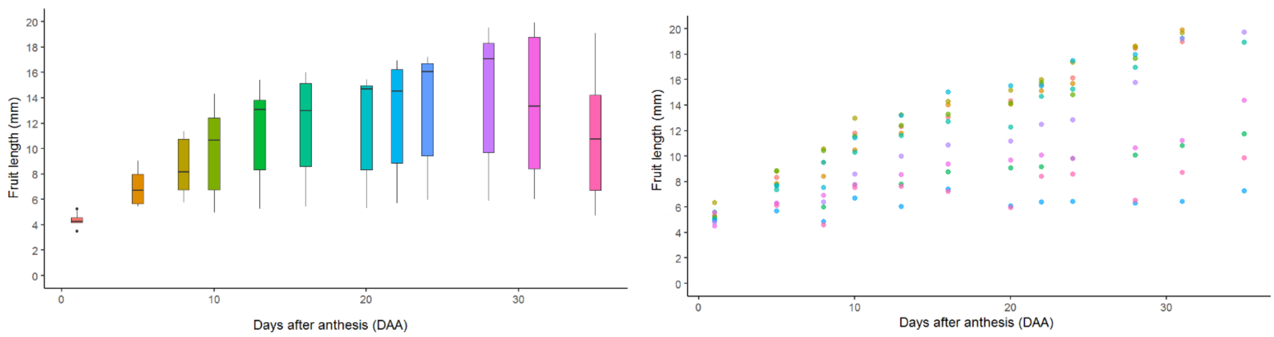

Figure 3 - Scatter plot and boxplot of 'Choctaw' blackberry length data over time.

The increase in data variability can be considered an indication of the need to model data heterogeneity and add a random effect to maximum growth parameters of models under study(PINHEIRO; BATES, 2004) or also adjust data quantiles with non-linear growth models via quantile regression (OLIVEIRA A. C. R. et al. (2021)).

Figure 4 shows the confidence intervals (CI) individually for each fruit. For CI that correspond to the adjustment of the Brody + Logistic and Double Logistic fixed models, several fruits did not have individual adjustments with convergence. For fruits in which it was not possible to obtain estimates, it was not possible to draw conclusions regarding parameters with need to include a random effect. In the other adjustments (Gompertz and Logistic), there was convergence of all individual adjustments and the assumption that in parameters β_1 and β_2 it is possible to add a random effect due to the non-overlapping of some of CIs (MEDEIROS S. D. S. et al. (2020), WYZYKIOWSKI et al (2015) and PINHEIRO; BATES (2004)).

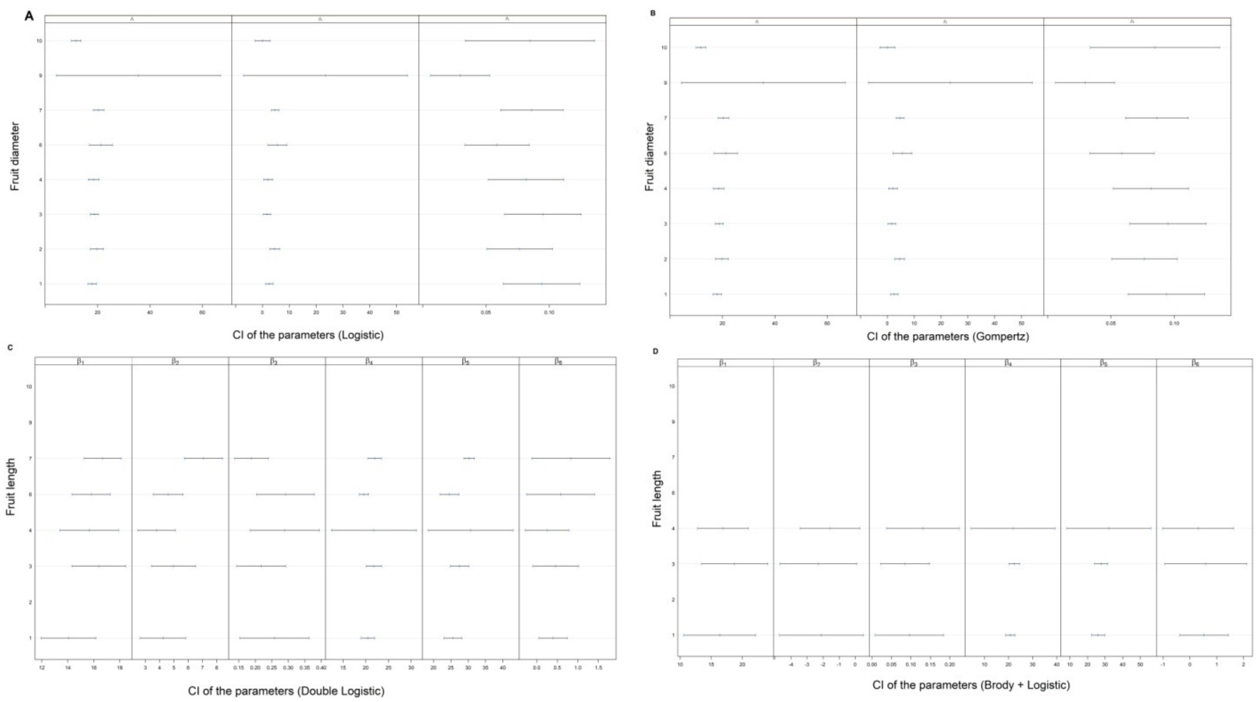

Figure 4 - Graphic representation of 95% confidence intervals of parameters of Logistic (A) and Gompertz (B) models for diameter growth and Logistic + Logistic (C) and Brody+Logistic (D) models for 'Choctaw' blackberry fruit length.

Non-linear fixed models were adjusted for fruit diameter and length via ordinary least squares, since, as mentioned by Wyzykiowski et al., (2015) and Medeiros et al., (2020), the first adjustment is performed considering that the residual vectors are independent, identically distributed by a zero mean normal and constant variance. After the adjustment, considering that all assumptions were met, the analysis of residuals is necessary to prove or not the hypotheses of the vector of residues.

The results of normality (SW), homoscedasticity (BP) and independence (DW) tests are shown in Table 3, and for fixed-effect models, it was possible to observe that all models had breach of normality assumptions, homogeneous and independent variances (p -value < 0.05), with graphic analyses (Figures 5, 6, 7 and 8), corroborating the tests adopted. With the breach of assumptions, modeling via mixed models, in addition to adding random effects to parameters, the incorporation of the first-order autoregressive term and heterogeneity modeling was necessary, according to Fernandes et al. (2014), Frühauf et al. (2022), Medeiros et al. (2020), Ribeiro et al. (2018a), Ribeiro et al. (2018b), Silva et al. (2020), Silva et al. (2021a), Silva et al. (2021b) and Wyzykiowski et al. (2015).

Table 3 – P-value of Shapiro-Wilk (SW), Breusch-Pagan (BP) and Durbin-Watson (DW) tests for the Logistic, Gompertz, Brody + Logistic and Double Logistic models of fixed effects adjusted to growth data of 'Choctaw' blackberry in relation to diameter and length.

Characteristic	Adjusted model	SW	BP	DW
Diameter	Gompertz	<0.001	<0.001	0.031
	Logistic	<0.001	<0.001	0.023
Length	Brody + Logistic	<0.001	<0.001	0.012
	Logistic + Logistic	<0.001	<0.001	0.026

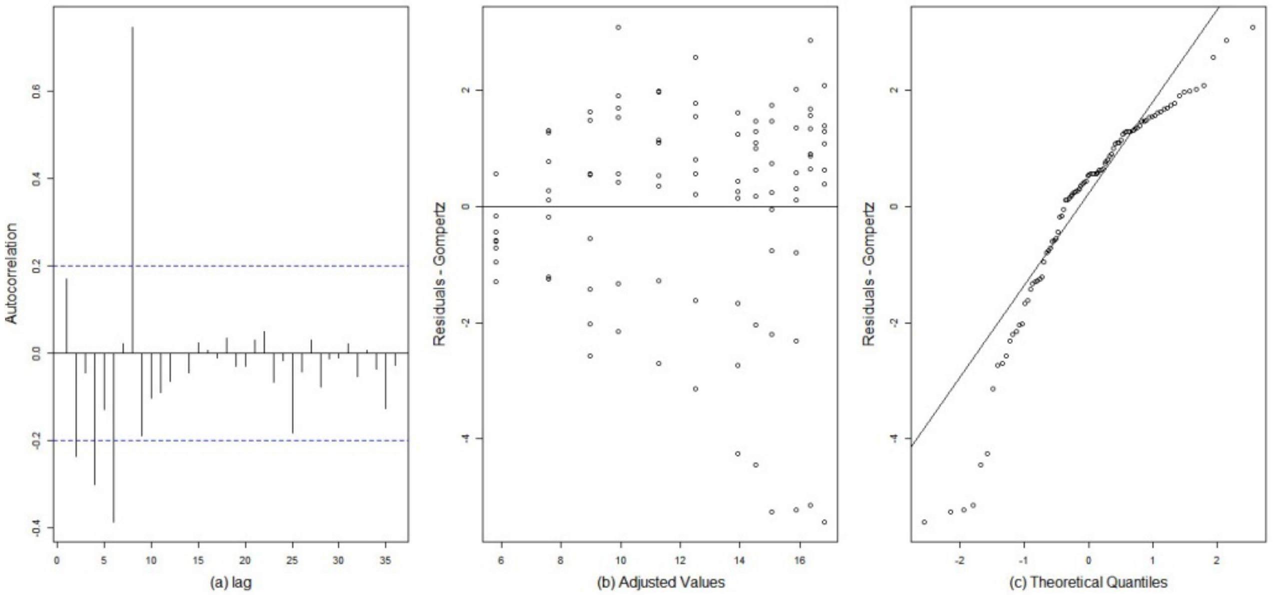

Figure 5 - Residual analysis for diameter growth, where (a) represents the autocorrelation graph by lags, (b) represents the adjusted values in relation to residuals and (c) residual values in relation to the theoretical quantiles for the simple Gompertz model.

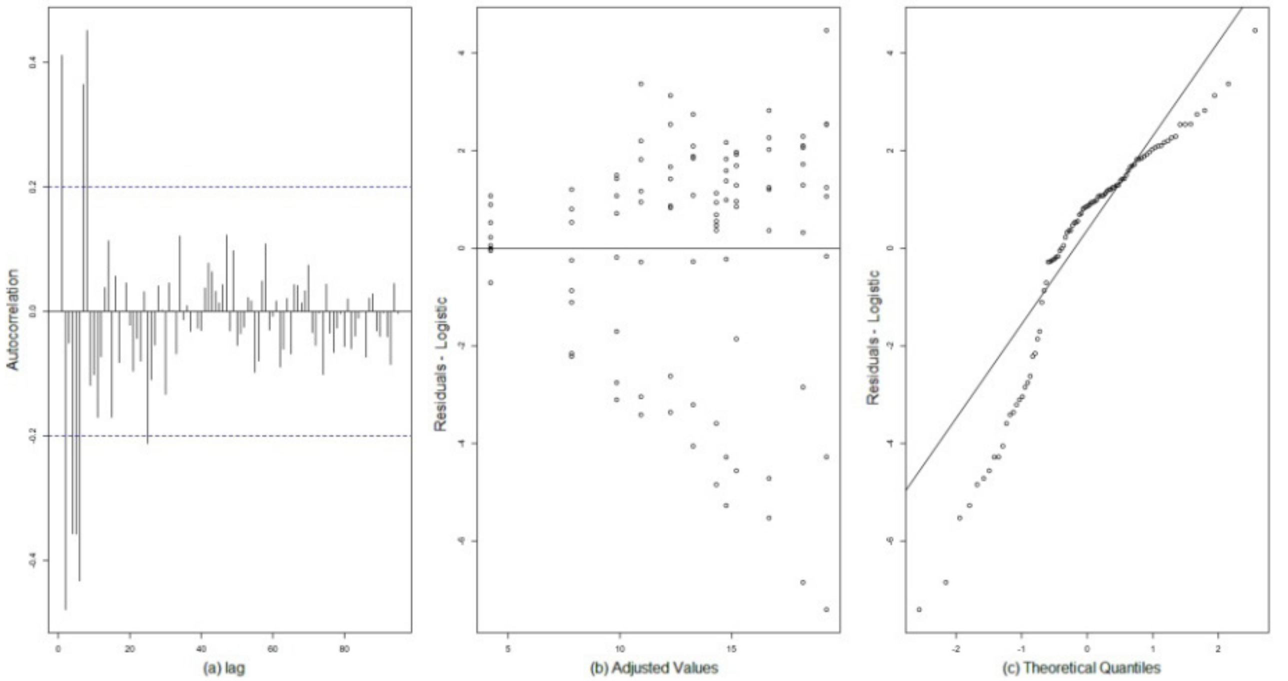

Figure 6 - Residual analysis for diameter growth, where (a) represents the autocorrelation graph by lags, (b) represents the adjusted values in relation to residuals and (c) residual values in relation to the theoretical quantiles for the simple logistic model.

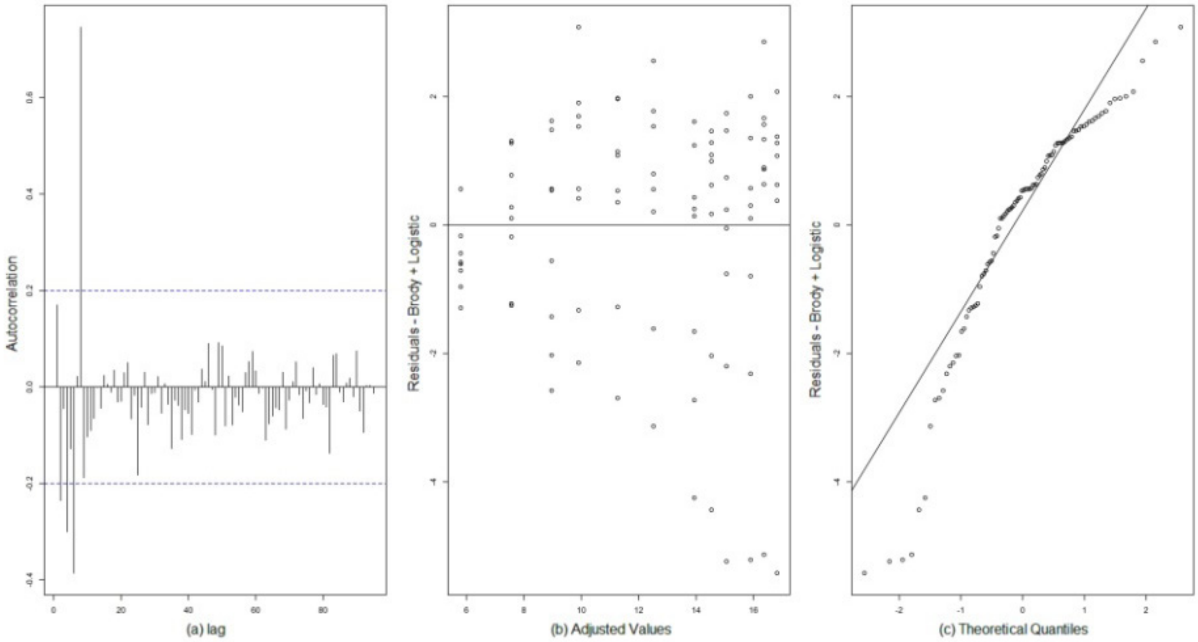

Figure 7 - Residual analysis for length growth, where (a) represents the autocorrelation graph by lags, (b) represents the adjusted values in relation to residuals and (c) residual values in relation to the theoretical quantiles for the Brody + Logistic model.

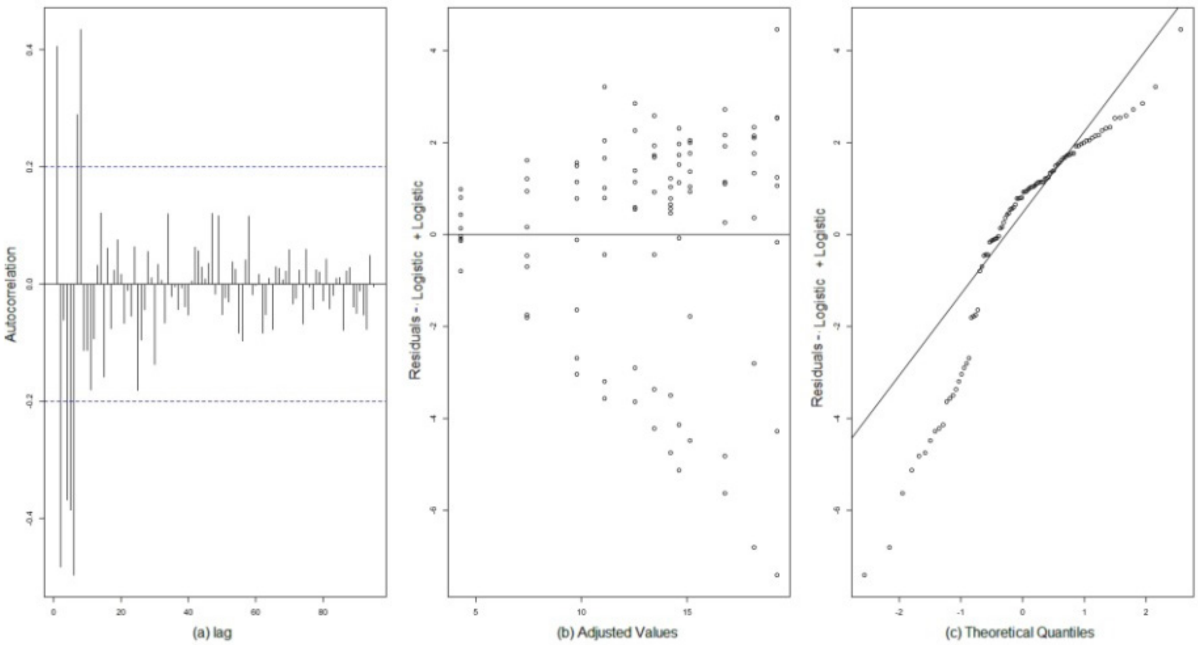

Figure 8 - Residual analysis for length growth, where (a) represents the autocorrelation graph by lags, (b) represents the adjusted values in relation to residuals and (c) the residual values in relation to the theoretical quantiles for the Logistic + Logistic model.

When evaluating the parametric non-linearity (c^{θ}) of Gompertz, Logistic, Double Logistic and Brody + Logistic models, all non-linearity values were less than 1, being considered that the models are adequate to describe the functional relationship between time and diameter and time and length (SARI et al. (2019a)).

Table 4 shows the fit quality evaluators of models for models that had convergence with the inclusion of random effects to parameters, where all had the inclusion of the first-order autoregressive term and some combinations with the varPower and varExp structure.

Table 4 - Criteria to assess the fit quality: adjusted determination coefficient (R^2_{aj}), Akaike information criterion (AICc), and the likelihood ratio test (TRV), to compare Logistic, Gompertz, Double Logistic and Brody + Logistic models with growth data from 'Choctaw' blackberry fruits in relation to diameter and length.

Characteristic	Adjusted model	Parameters	R^2_{aj}	AICc	TRV
Length	Brody + Logistic	a and a1	0.98	437.15	-208.21
		a and a1*	**0.98**	**435.54**	**-206.43**
		a and a1**	0.98	437.22	-207.27
	Logistic + Logistic	a and a1	0.98	426.65	-202.99
		a and a1*	**0.98**	**425.41**	**-201.37**
		a and a1**	0.98	427.37	-202.34
Diameter	Gompertz	a	1.00	275.18	-132.46
		k	1.00	277.37	-133.55
		a and b	1.00	269.97	-127.85
		a and b*	**0.97**	**263.87**	**-124.80**
		a and b**	0.97	265.92	-125.83
	Logistic	a	1.00	289	-139.36
		k	0.8	405.52	-197.63
		a and b	1.00	282.4	-134.06
		a and k	1.00	284.25	-142.32
		a and b*	**1.00**	**269.68**	**-127.71**
		a and b**	1.00	271.17	-128.45

* varPower; ** varExp;

When comparing the results for fruit length with Brody + Logistic and Logistic + Logistic models with the varPower weight matrix, the highest adjusted determination coefficient and likelihood ratio test values, as well as the lowest AICc values were found. In the study by Silva et al. (2020), the Logistic + Logistic fixed-effects model was found to be the best model for 'Choctaw' blackberry data.

For diameter results, similarly to length, models adjusted with the inclusion of the varPower weight matrix were those with the best quality indicators for Gompertz and Logistic models. For length, the Logistic + Logistic model showed the best adjustment, as found in the study by Silva et al. (2020) and for diameter, the logistic model, the same model found by Muianga et al. (2016) for cashew data.

When defining the two best models for fruit diameter and length, the analysis of their residues was carried out and results are shown in Figures 9 and 10.

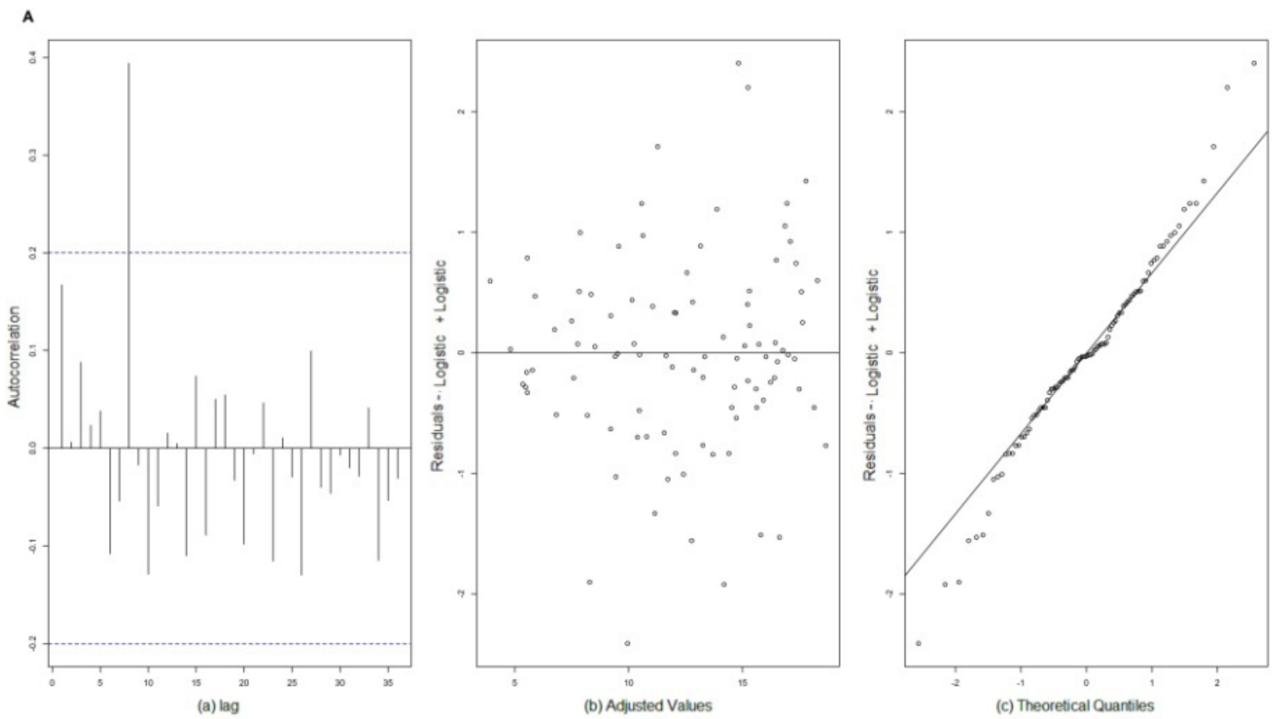

Figure 9A. Residual analysis for length growth, where (a) represents the autocorrelation graph by lags. Residual analysis for length growth, where (b) represents the adjusted values in relation to residuals (c) residual values in relation to the theoretical quantiles for Logistic + Logistic models with random effect on β1 and β4 with varPower.

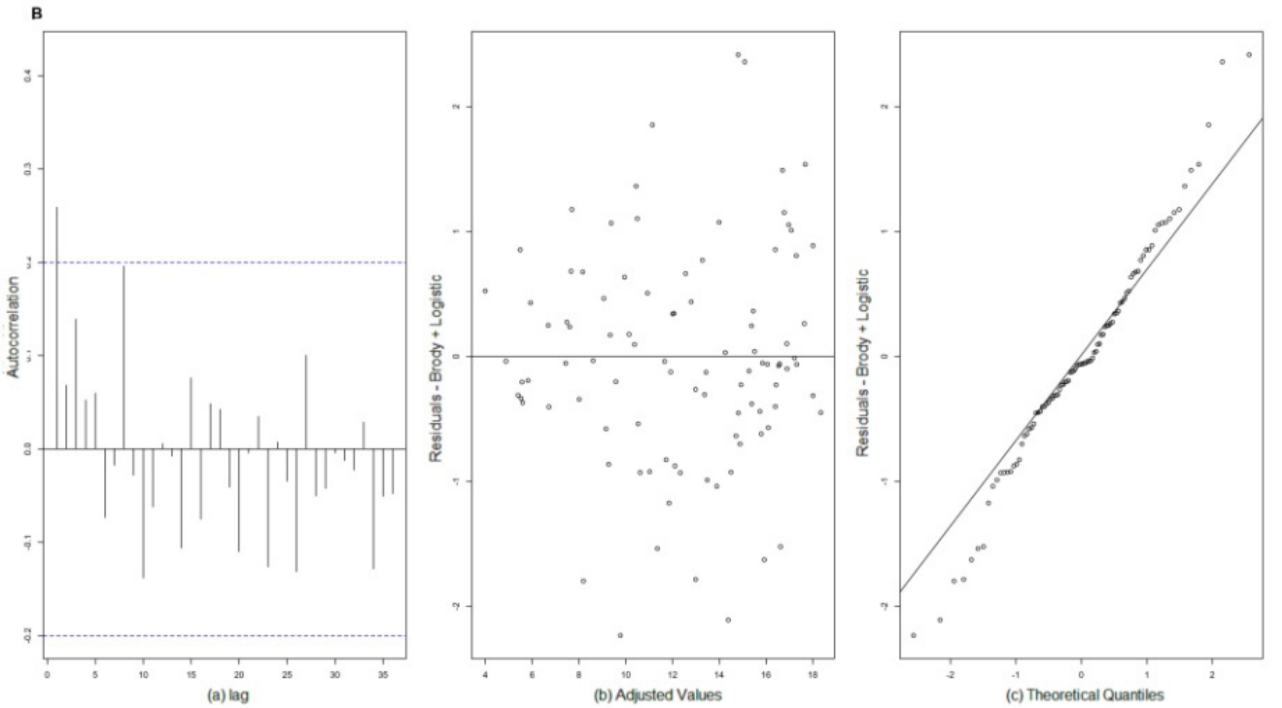

Figure 9B. Residual analysis for length growth, where (a) represents the autocorrelation graph by lags. Residual analysis for length growth, where (b) represents the adjusted values in relation to residuals (c) residual values in relation to the theoretical quantiles for Brody + Logistic models with random effect on β1 and β4 with varPower.

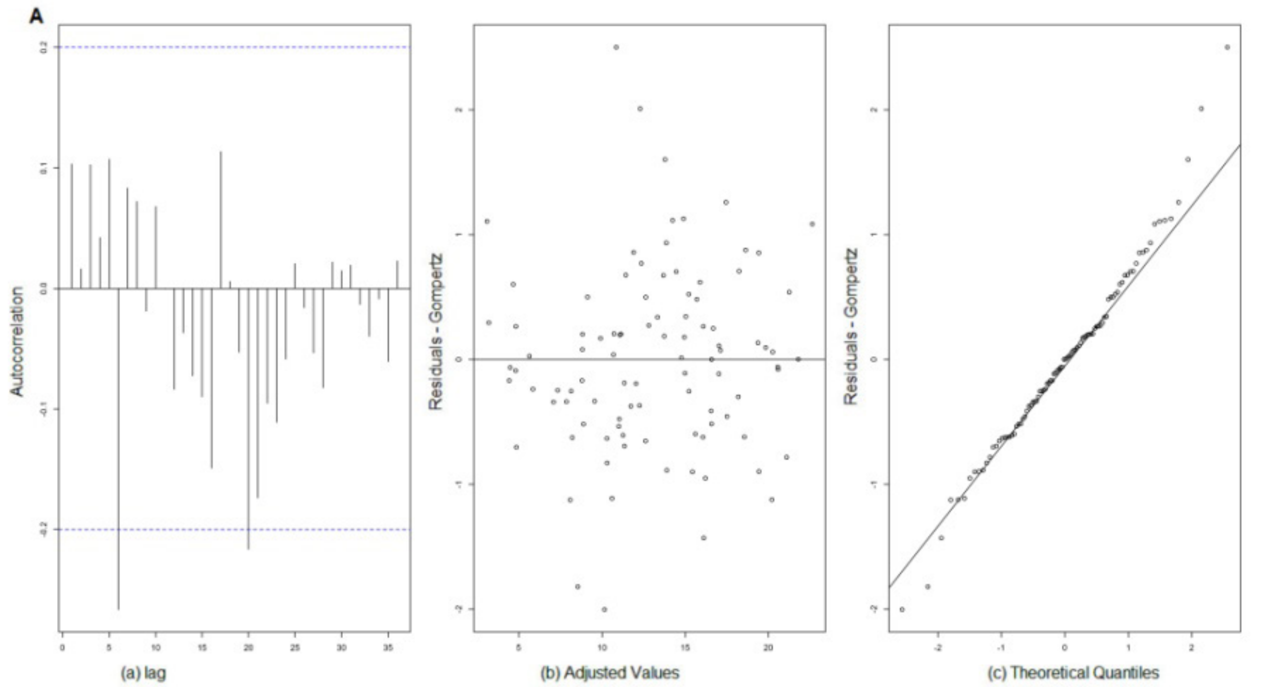

Figure 10A. Residual analysis for length growth, where (a) represents the autocorrelation graph by lags. Residual analysis for length growth, where (b) represents the adjusted values in relation to residuals and (c) residual values in relation to the theoretical quantiles for Gompertz models with random effect on β1 and β2 with varPower.

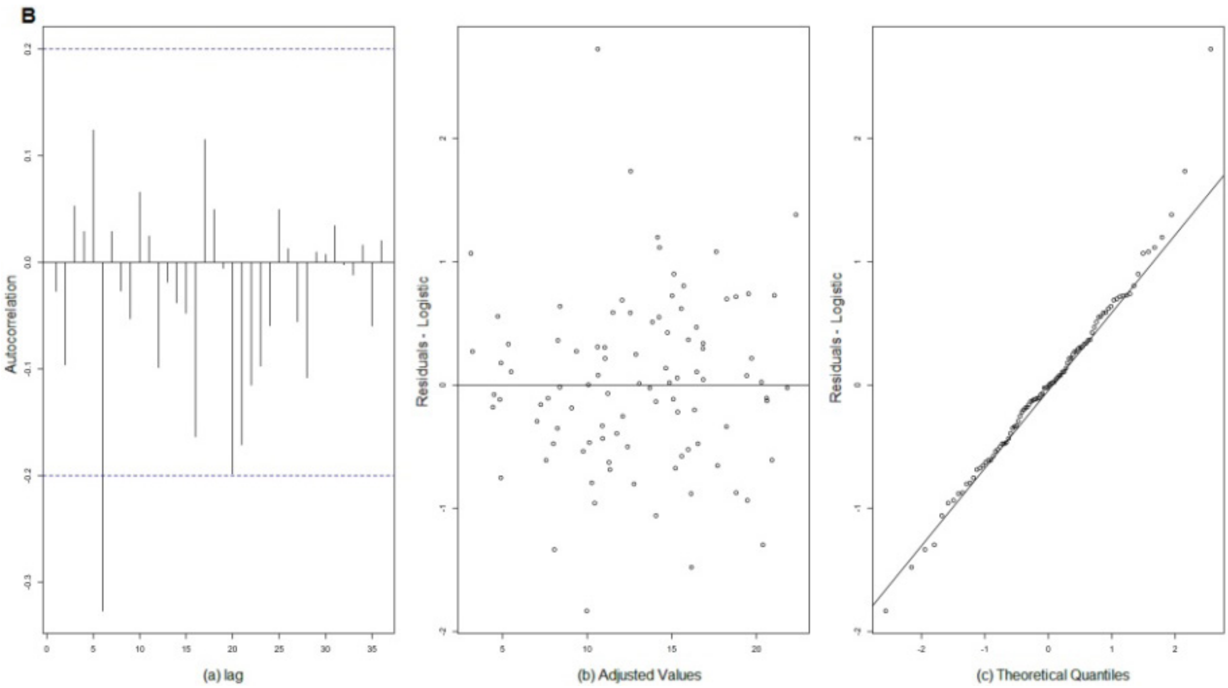

Figure 10B. Residual analysis for length growth, where (a) represents the autocorrelation graph by lags. Residual analysis for length growth, where (b) represents the adjusted values in relation to residuals and (c) residual values in relation to the theoretical quantiles for Logistic models with random effect on β1 and β2 with varPower.

As could be observed, for the two best models, for length and diameter, the graphic analysis of lags (a), adjusted values *vs* residuals (b) and the graph of theoretical quantiles vs residuals (c) that the inclusion in the weight matrix (varPower) and first-order residual autocorrelation were efficient to make the vector of residuals meet the initial assumptions of the statistical modeling and thus be able to have more reliable inferences of the estimated parameters (FRÜHALF A. C. et. al., 2020; JANE et al., 2020; PRADO et al., 2020).

Table 5 shows the estimates of adjusted models with the best fit quality values for the length and diameter of blackberry fruits. The maximum expected diameter growth was 17.24 mm, a statistically similar value in relation to that found by Silva et al. (2020), who found value of 17.11 mm, thus showing that the maximum expected diameter growth of 'Choctaw' blackberry fruits is approximately 17.00 mm, with the inflection point occurring before the expected half of 7.125 days.

Table 5 - Estimates of parameters for Brody + Logistic, Logistic + Logistic, Gompertz and Logistic models applied to the growth data of 'Choctaw' blackberry fruits in relation to diameter and length.

Characteristic	Adjusted model	Parameters	β_1	β_2	β_3	β_4	β_5	β_6
Length	Brody + Logistic	a and a1 (varPower)	17.24*	-2.75	0.07*	20.16*	29.24*	0.66*
	Logistic + Logistic	a and a1 (varPower)	14.69*	4.91*	0.21*	19.54*	28.69*	0.43*
Diameter	Gompertz	a and b (varPower)	17.87*	3.26*	0.08*	---	---	---
	Logistic	a and b (varPower)	17.11*	7.15*	0.12*	---	---	---

*Significant at 5% significance.

As for length, the asymptotic value of the first growth phase occurred when fruits had average of 17.24 mm and maximum expected growth of 20.16 mm. The parameter linked to the growth rate in the first phase was greater than in the second, which was also observed by Silva et al. (2020) and Fernandes et al (2022).

Analyzing results found in Table 5, it was possible to observe that all parameters were significant to explain the non-linear relationship between variable time and fruit diameter and length. According to Silva et al. (2020) and Fernandes et al. (2022), both models are indicated to describe such relationship, that is, parameters were significant, with vector of residuals met.

With adjustments of the best model for length and diameter, Figure 11 shows how, over the days, fruit growth occurred individually and in the random and fixed-effect models for Logistic + Logistic (length) and Logistic (diameter) models. As demonstrated in steps so far, in this work, the adjustment of models, adding random effects to parameters, better explained the existing data variance.

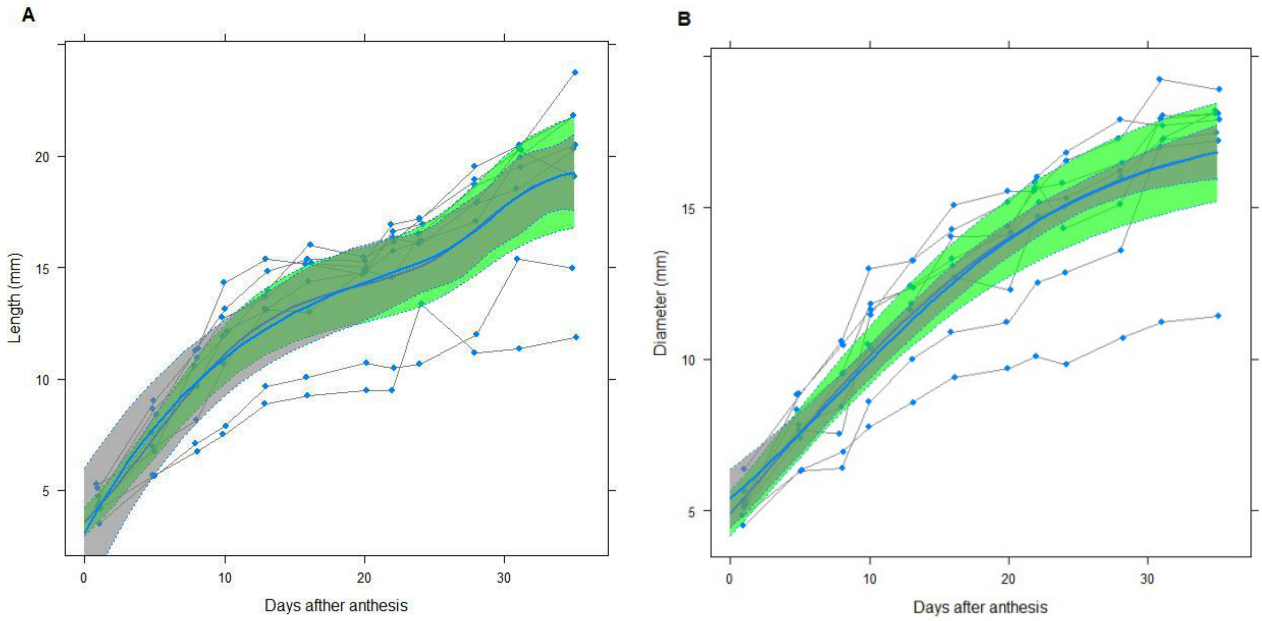

Figure 11 - Adjusted curves of fixed-effect and random-effect models for the Logistic + Logistic model (A) and the Logistic model (B) referring to ‘Choctaw’ blackberry fruits.

Analyzing both graphs, it was possible to observe that the range of the curve estimated for random-effect models better shows the variation in the growth of individuals, both at the beginning and at the end of the productive cycle (Figure 11). Analyzing graphs, it is clear that the confidence interval for the adjustment of random effects (in green) does not remain constant over time, being, at the beginning, more concentrated and, at the end, more dispersed, corroborating results shown in Figures 2, 3 and Table 2; on the other hand, the fixed-effect model remains constant over time.

Analyzing the present results, in Figure 11, considering some parameters with random effects for the growth of blackberry fruits (diameter and length) is necessary due to the results found so far and, according to Pinheiro and Bates (2004), ignoring this finding can lead to mistaken estimates of inferences in relation to models.

Figure 12 shows graphs of growth rates for length (on the left) and diameter (on the right) of Logistic + Logistic and Simple Logistic models, respectively. The abscissa and ordinate estimates of the maximum acceleration point (pma), inflection point (pi), maximum deceleration point (pmd) and asymptotic deceleration point (pda) were taken from studies by Silva et al. (2021b) and correspond to critical points. However, Silva et al. (2021b) estimated the points for simple sigmoid models (Gompertz and Logistic); however, as double models are the sum of models, estimates can be used both to estimate a simple development cycle and a double development cycle.

When analyzing results in Figure 12, it was observed that the inflection point occurred exactly at the peak of each curve, that is, for the simple Logistic model with 8.55 mm and at 7.15 DAA, for the double Logistic model, for the first growth phase was 7.35 mm and 4.91 DAA, while in the second phase, 9.77 mm and 28.69 DAA. The asymptotic deceleration point was 15.54 and 14.69 for diameter and length, respectively, and Michan and Pinho (2014) reported that this value corresponds to approximately 90% of the maximum expected fruit growth and can therefore be used as a parameter for fruit harvest.

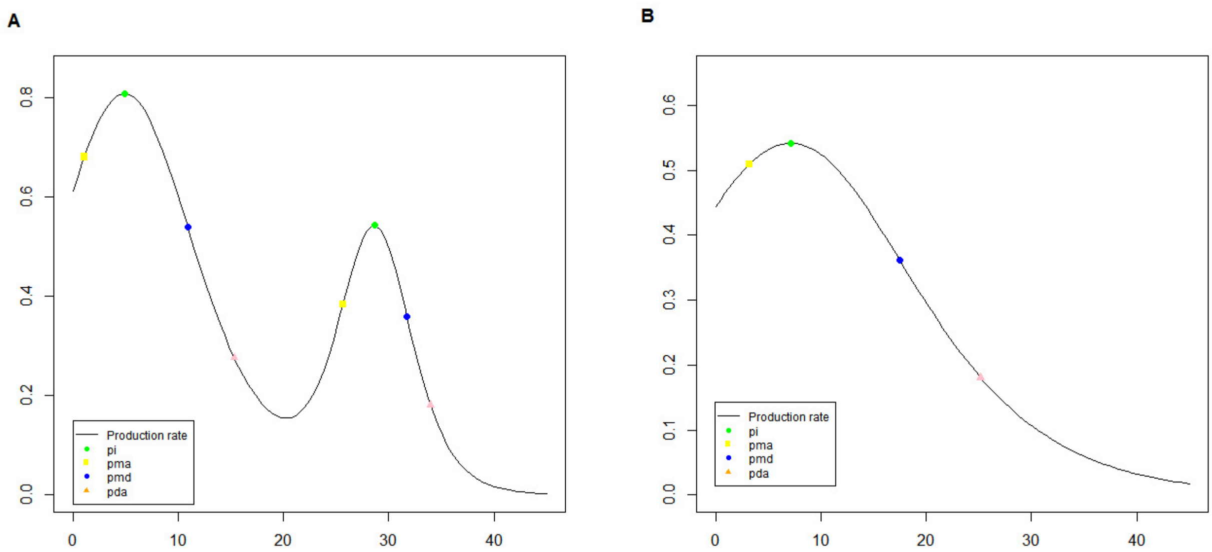

Figure 12 - Growth rate curves for Logistic + Logistic (A) and Logistic (B) models with maximum acceleration points (PMA), inflection point (PI), maximum deceleration point (PMD) and asymptotic deceleration point (PDA) for both length development stages and for the single diameter development stage.

Conclusions

By the fit quality evaluators, the choice of models that best adjusted to data were the double Logistic and the simple Logistic, respectively, for length and diameter. In all models, it was necessary to incorporate the varPower function and the first-order residual autocorrelation. The incorporation of the random effect was indicated for the Double Logistic model in parameters β_1 and β_4 and in the Simple Logistic model in parameters β_1 and β_2.

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